Clemson University
School of Mathematical & Statistical Sciences

MATH 8530-001, Linear Algebra / Matrix Analysis, Fall 2020
Syllabus

"This is the plan, up until it is no longer the plan."
–Anonymous university administrator (not at CU)

Instructor: Dr. Matthew Macauley

Course Description: This course will be a comprehensive survey of finite dimensional vector spaces and linear mappings at the graduate level. The goals are two-fold – to prepare students for the linear algebra part of the prelim, and to build up a solid linear algebra foundation necessary for other classes and research, as linear algebra is a beautiful subject that appears in nearly all areas of mathematics, *especially* in applied fields such as statistics, operations research, and computational mathematics. Some advanced linear algebra classes, especially those geared to engineers and often called "matrix analysis," focus on computations involving matrices and under-emphasize the theory of vector spaces. Yet many other treatments take the opposite approach by presenting the theory of modules, which is too far detached from most applications that an applied mathematician, statistician, or optimizer will need. I will try to take a "Goldilocks" approach, and emphasize the theory of vector spaces (not modules), while being grounded in applications. The book that I will follow, despite being quite theoretical, was written by a foremost expert on computational PDEs, containing that material that thought was most important for his applied mathematics graduate students to learn over the course of a year-long class.

Prerequisite: Undergraduate linear algebra class (Math 3110 at Clemson).

Communication Strategy: Email (**macaule@clemson.edu**) is the best way to reach me. I will check it *at least* every few hours during the hours of 8am–9pm, seven days a week.

If you send me an email and do not get a reply by the time you go to bed, please re-send it, as that is my mistake. Just click "Reply" and "Send"; no need to explain.

Students are responsible for checking their Clemson email regularly, as that address will be the one subscribed to the class email list. I encourage all of us to be on a first-name basis.

I am required to include my office phone (656-1838), but strongly recommend using email instead.

Happy Hour: Several evenings a week at 7pm, you are invited to join me and your classmates on Zoom for an Adult Beverage[1], company, and office hours. I'll stick around to answer questions as long as there are some. If it's 7:15pm and nobody is there, I will log off. The exact days will be announced in the beginning of the week, and will depend on when homework is due, in order to optimize the utility of this time.

Useful websites:

Course webpage: `http://www.math.clemson.edu/~macaule/classes/f20_math8530/` (all relevant links posted here)

Canvas: `https://www.clemson.edu/canvas/` (will be only used to submit homework and exams)

[1] For me, this means drinks like **LaCroix** or **Kombucha**, which are *very unpopular* among kids.

Texts: You do not need to buy a book for this class. In lecture, I will roughly follow the first 9 chapters of Peter Lax's book *Linear Algebra and its Applications*, though I will use Paul Halmos' classic *Finite Dimensional Vector Spaces* when we cover determinants. I will also occasionally draw from Gil Strang's book. For a nice freely available book that covers the same material we are, I recommend:

J. Gallier and J. Quaintance, *Algebra, Topology, Differential Calculus, and Optimization Theory for Computer Science and Machine Learning*, Part I, 2019, freely available online at `http://www.cis.upenn.edu/~jean/gbooks/geomath.html`

Schedule: This course will be conducted in an online format. I am still deciding on the proportion of live vs. pre-recorded lectures. It depends on how things go and what the class prefers. Most likely, many lectures will be posted to YouTube and given asynchronously, and we will utilize class time for questions, discussion, and to work on homework. Naturally, this is all subject to change. The unique advantage of asynchronous lectures, which is especially true in graduate math classes, is the crucial ability to pause and/or re-watch any statements that are not 100% clear.

Required technology:

A computer and reliable internet connection.

A free Zoom account, and a video camera that allows you to be recorded over Zoom (for exam proctoring).

A smartphone scanning app. There are many free apps, such as CamScanner or Adobe Scan. If you do not have a smartphone, a traditional scanner will suffice, but a smarphone app is prefered.

Access to a program that runs LaTeX. I recommend `overleaf.com`.

Zoom Info: Please turn your camera on for all Zoom meetings, if possible. Note that this is required for taking exams.

The URL will be the same for all classes and Happy Hours, and will be emailed to the class list. I am also available to meet by appointment, if desired. In that case, email me and include block(s) of time in which you are available. Please let me know in advance if you want any meeting to be private, like if you want to discuss your grade. In that case, I will use a different Zoom meeting.

The class Zoom URL will be available 24 hours a day, 7 days a week. Feel free to meet up with your classmates on there anytime you would like, especially if you want to collaboriate on the homework.

There will be a different URL, with different settings, for exams to be taken over Zoom.

If your internet goes out during a Zoom meeting, Zoom should automatically reconnect when it comes back. This happens to me several times a day, for a few minutes at a time, because AT&T is terrible. It will inevitably happen several times during class throughout the semester. In this case, please stick around, and consider this to be an unexpected 1–2 minute break from class.

If I do not show up by the beginning of class, check your email. If you have not heard anything from me by 9:10, you may assume that class has been canceled.

Homework: Weekly homework assignments are posted on the course webpage. *Submitted assignments must be typeset with LaTeX.* Working together is encouraged, but everyone must do their own work, and *collaborators must be cited.*

Homework assignments are due at 11:59pm. Assignments can be submitted multiple times; only the last submission will be graded. There is a 3-hour grace period for deadlines, meaning that there is

no penalty for assignments submitted within 3 hours of the deadline. Canvas will *not* accept any submissions after that point.

Exams: There will be two 50-minute midterm exams during the semester and a cumulative 2.5-hour final exam. They will be accessible on Canvas, and I will proctor all exams over Zoom. You must provide consent to having the meeting recorded.

Exam checklist (things to bring):

- Plenty of blank scratch paper and pens or pencils.
- Smartphone (for scaning your exam when you finish).

Exam rules:

- Before beginning the exam, you must do a "room scan" with your camera, and also verify that all of the paper you brought is indeed blank.
- You must share your video for the entire duration of the exam.
- The camera must be far enough away so I can see your hands and paper at all times. That is, I must be able to verify that you are not using a phone or computer.
- When you are finished, send me a private Zoom Chat to let me know, and then scan and email your exam to me while still on camera. It must be scanned in one multi-page pdf document, and *not* multiple individual one-page documents.
- Before *and* after submitting, double-check to make sure that the scanned file is (i) fully legible, (ii) complete, and (iii) the correct file.

It is strongly recommended that you practice with your smartphone scanning app *before* the exam.

Grading: Your final grade will be computed as follows:

Homework	25%
Midterm 1	25%
Midterm 2	25%
Final Exam	50%

I will drop either your lowest midterm grade, OR half of the weight of the final exam; whichever is lowest. Also, if you get an unambiguous **A** on the final exam, then you will get an **A** in the course, *assuming you have a passing grade on the homework.*

I do *not* grade using arbitrary round number cut-offs. I generally err on the side of having difficult exams (e.g., sometimes the median score is in the 50–60% range). This spreads out the distribution, and the scores generally fall into visually clear distinct "clusters" with large gaps between them. Roughly speaking, an **A** is the "very good" cluster, a **B** is statistically significantly below that, and a **C** is for outliers on the low end. I will use + and - grades for borderline cases, or for a bi-modal or very wide cluster. Though past performance is not an indicator of future results, I have never given fewer than 50% **A**s in a graduate class.

Because of the aforementioned policies, the automatically calculated numeric grade that you see in Canvas, which is the average of every weighted graded submission, is NOT an accurate indicator of your grade. At any point in time during the class, I would be happy to give you a ballpark estimate of how you are doing.

Key Dates

Aug 19 (Wed)	Classes begin
Aug 25 (Tue)	Last day to register or add a class
Sep 1 (Tue)	Last day to drop a class or withdraw from the University without a **W** grade
Oct 23 (Fri)	Last day to drop a class or withdraw from the University without final grades
Nov 2–3 (M,Tu)	Fall break
Nov 25–27 (W–F)	Thanksgiving break
July 29 (Wed)	Last day of class
Dec 11 (Fri)	Final Exam, 8:00–10:30am

Student Learning Outcomes: Upon successful completion of MATH 8530, students will be able to

Demonstrate a solid understanding of the theory of vector spaces and linear maps.

Abstract skills and knowledge learned in undergraduate linear algebra to a more general setting.

Apply concepts from linear algebra to problems in applied mathematics and statistics.

Read, write, and critique rigorous mathematical proofs on topics in linear algebra.

Develop good mathematical writing skills. Important aspects of this are *accuracy*, *clarity*, and *conciseness*.

"No exceptions": In any class syllabus, no matter how they are worded, policies and phrases like "no exceptions", "no make-ups", etc. are *never* actually what they sound, and this is especially true this semester. Things happen, from natural distasters (hurricanes, tornados), to human disasters (9/11, school shootings), to personal and family tragedies, to health emergencies (COVID, auto accidents, hospitalizations). This does not mean that any exception or extension will be granted, but I will do my best to be reasonable, fair, and accommodating.

Make-Up Policy: I will drop your lowest midterm, which means that if you miss a midterm, then your final exam grade will replace it. The homework deadlines will not be extended for individual students, and assigned homework must be turned in by the deadline. PLAN AHEAD: If you submit assignments minutes before the deadline, you take the risk of bad luck, e.g., a power outage, computer freeze or crash, personal emergency, zombie attack, etc., that could make you miss the deadline.

By default, any exam that was scheduled at the time of a class cancellation due to power outage / inclement weather will be given at the next class meeting. Any extension or postponement of assignments or exams must be granted by me via email or Canvas within 24 hours of the weather-related cancellation.

Special Accommodations: Clemson University values the diversity of our student body as a strength and a critical component of our dynamic community. Students with disabilities or temporary injuries/conditions may require accommodations due to barriers in the structure of facilities, course design, technology used for curricular purposes, or other campus resources. Students who experience a barrier to full access to a class should let the instructor know, and make an appointment to meet with a staff member in Student Accessibility Services as soon as possible. You can make an appointment by calling 864-656-6848 or by emailing studentaccess@lists.clemson.edu. Students who receive Academic Access Letters are strongly encouraged to request, obtain and present these to their instructors as early in the semester as possible so that accommodations can be made in a timely manner. It is the student's responsibility to follow this process each semester. You can access further information here: `http://www.clemson.edu/campus-life/campus-services/sds/`.

COVID-19: Most of the following will not apply to our online class, but I am required to include it. While on campus, face coverings are required in all buildings and classrooms. Face coverings are also required in outdoor spaces where physical distance cannot be guaranteed. Please be familiar with the additional information on the Healthy Clemson website (`https://www.clemson.edu/coronavirus/index.html`), such as the use of disinfectant wipes for in-person classes.

If a student does not have a face covering or refuses to wear an approved face covering without valid documented accommodation, *I will ask the student to leave the academic space and will report the student's actions to the Office of Community & Ethical Standards as a violation of the Student Code of Conduct*. If the student's actions disrupt the class to the extent that an immediate response is needed, I may call the Clemson University Police Department at 656-2222.

Mental health: Your mental health is important to me, and I am always available to talk. Please don't hesitate to reach out. We're in this together, and all of us are strugling in some regards, myself included.

Title IX Policy: Clemson University is committed to a policy of equal opportunity for all persons and does not discriminate on the basis of race, color, religion, sex, sexual orientation, gender, pregnancy, national origin, age, disability, veterans status, genetic information or protected activity (e.g., opposition to prohibited discrimination or participation in any complaint process, etc.) in employment, educational programs and activities, admissions and financial aid. This includes a prohibition against sexual harassment and sexual violence as mandated by Title IX of the Education Amendments of 1972.

The University is committed to combatting sexual discrimination including sexual harassment and sexual violence. As a result, you should know that University faculty and staff members who work directly with students are required to report any instances of sexual harassment and sexual violence, to the Universitys Title IX Coordinator. What this means is that as your professor, I am required to report any incidents of sexual harassment, sexual violence or misconduct, stalking, domestic and/or relationship violence that are directly reported to me, or of which I am somehow made aware.

There are two important exceptions to this requirement about which you should be aware:

Confidential Resources and facilitators of sexual awareness programs such as "Take Back the Night and Aspire to be Well" when acting in those capacities, are not required to report incidents of sexual discrimination.

Another important exception to the reporting requirement exists for academic work. Disclosures about sexual harassment, sexual violence, stalking, domestic and/or relationship violence that are shared as part of an academic project, a research project, classroom discussion, or course assignment, are not required to be disclosed to the Universitys Title IX Coordinator.

This policy is at `http://www.clemson.edu/campus-life/campus-services/access/title-ix/`. Alesia Smith is the Executive Director for Equity Compliance and the Title IX Coordinator. Her office is at 223 Holtzendorff Hall, phone number is 864.656.3181, and email address is alesias@clemson.edu.

Academic Integrity: As members of the Clemson University community, we have inherited Thomas Green Clemson's vision of this institution as a 'high seminary of learning'. Fundamental to this vision is a mutual commitment to truthfulness, honor, and responsibility, without which we cannot earn the trust and respect of others. Furthermore, we recognize that academic dishonesty detracts from the value of a Clemson degree. Therefore, we shall not tolerate lying, cheating, or stealing in any form.

Topics: We will cover the following topics, roughly in this order.

Section 1: Linear algebra fundamentals [chapter 1 of Lax]

- Vector spaces
- Linear maps
- Subspaces
- Spanning, independence, bases, and dimension
- Complementary subspaces and direct sums
- Direct products
- Finite vs. infinite products
- Quotient spaces
- Application: solving linear ODEs

Section 2: Duality [chapter 2 of Lax]

- Linear functionals & the dual space
- Scalar product notation
- Annihilators
- Double duals

Section 3: Linear mappings [chapter 3 of Lax]

- Range & nullspace
- Rank-nullity theorem
- Appliation: systems of linear equations
- Application: polynomial interpolation
- Application: average values of polynomials over intervals
- Application: numerial solutions to Laplace's equation (finite differences)
- Algebra of linear mappings
- Transposes (as mappings between dual spaces)

Section 4: Matrices [chapter 4 of Lax]

- How a choice of basis determines the matrix of a linear map
- 4 ways to multiply matrices (row-by-cols, by rows, by cols, col-by-rows)
- The matrix of a transpose map
- Column rank & row rank
- Change of basis & similar matrices
- Systems of equations and Gaussian elimination

Section 5: Determinant and trace [Halmos' book + Strang's book + chapter 5 of Lax]

- Geometric idea of determinant
- Permutations & discriminant
- Multilinear forms

- Symmetric, skew-symmetric, and alternating k-linear forms
- The vector space of alternating n-linear forms is 1-dimensional
- Basis-free definition & universal property of the determinant
- Determinants & matrices (Laplace expansion)
- Cramer's rule
- Trace

Section 6: Spectral theory [chapter 6 of Lax + appendix 15 of Lax + supplimental]

- Eigenvectors & eigenvalues
- Distinct eigenvalues lead to linearly independent eigenvectors
- Spectral mapping theorem: Eigenvalues of A vs. $q(A)$.
- Cayley-Hamilton theorem
- Algebraic multiplicity vs. geometric multiplicity of eigenvalues
- Generalized eigenvectors
- Spectral theorem: there is always a full set of generalized eigenvectors
- Minimal polynomials and Jordan canonical form
- Commuting maps and simulataneous diagonaliziability
- Application: systems of ODEs & matrix exponentials

Section 7: Euclidean structure (inner product spaces) [chapter 7 of Lax]

- Review of Euclidean structure (length, dot product, orthogonality, angles)
- Real inner product spaces
- Cauchy-Schwarz inequality
- Triangle inequality & Pythagorean theorem
- Orthonormal bases
- Gram-Schmidt process & QR-factorization
- Identification of a space with its dual
- Orthogonal complement & projections
- Adjoints
- Application: least squares
- Isometries & orthogonal matrices
- Norms of linear maps
- The subset of invertible maps is open
- Basic analysis review (convergence, Cauchy sequences, completeness, local compactness)
- An inner product spaces is locally compact iff it is finite-dimensional
- Complex inner product spaces
- Complex inner product, orthogonality, and unitary maps
- Application: Fourier series (real and complex)

Section 8: Self-adjoint mappings [chapter 8 of Lax]

- Decomposition of a linear map into a self-adjoint plus an anti-self-adjoint map
- Motivation: 2nd order Taylor approximations & the Hessian
- Quadratic forms
- Self-adjoint maps have real eigenvalues and a full set of orthonormal eigenvectors
- Projections onto eigenspaces and spectral resolutions
- Self-adjoint commuting maps have a common spectral resolution
- Anti-self-adjoint maps have purely imaginary eigenvalues and a full set of orthonormal vectors

- Normal maps
- Unitary maps
- The Rayleigh quotient & its critical points
- Minmax principle for the eigenvalues of a self-adjoint map
- Positive definite mappings and the generalized Reyleigh quotient
- Application to numerical linear algebra: 2nd order Taylor approximation of eigenvalues
- Properties of A^*A.

Section 9: Positive(-definite) mappings [chapter 10 of Lax]

- Tensor product of two vector spaces (4 different ways to think of it)
- Basic properties of positive and non-negative mappings
- A partial order on on the set of self-adjoint maps
- Symmetrized products
- Monotone matrix functions (MMFs)
- A functional analysis characterization of all MMFs
- Gram matrices & non-standard inner products
- Schur's theorem of positive matrices
- Singular value decomposition
- Right, left, and pseudo-inverses

Extra topics (since I'll probably finish a few days early) [misc. sources]

- Cyclic subspaces
- Companion matrices and rational canonical form
- What is a module? (A vector space over a ring)
- How Jordan & rational canonical form generalize from vector spaces to modules
- Avoidance of crossings (the space of singular self-adjoint mappings has co-dimension 2).