Lecture 4.1: Eigenvalues and eigenvectors

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Assumptions and definitions

Throughout, we will assume that A is an $n \times n$ matrix over K. Thus, it represents an endomorphism of a vector space $X \cong K^n$.

We will assume that K is algebraically closed, which means that every non-constant polynomial has a root in K.

The most common algebraically closed field is $K = \mathbb{C}$.

Definition

If $Av = \lambda v$ for some nonzero vector v and scalar $\lambda \in K$, then v is an eigenvector and λ is an eigenvalue.

Existence of eigenvectors

Proposition 4.1

A has an eigenvector.

An example

Remark

$A - \lambda I$ is noninvertible iff $\det(A - \lambda I) = 0$. That is, λ is an eigenvalue of A iff $\det(A - \lambda I) = 0$, and the corresponding eigenvector is any $v \neq 0$ in $N_{A-\lambda I}$.

Let's compute the eigenvalues and eigenvectors of $A = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$.

Linear independence of eigenvectors

Proposition 4.2

Eigenvectors of A corresponding to distinct eigenvalues are linearly independent.

Diagonalizability

Proposition 4.3

If X has a basis of eigenvectors of A, then A is similar to a diagonal matrix. We say that A is diagonalizable.