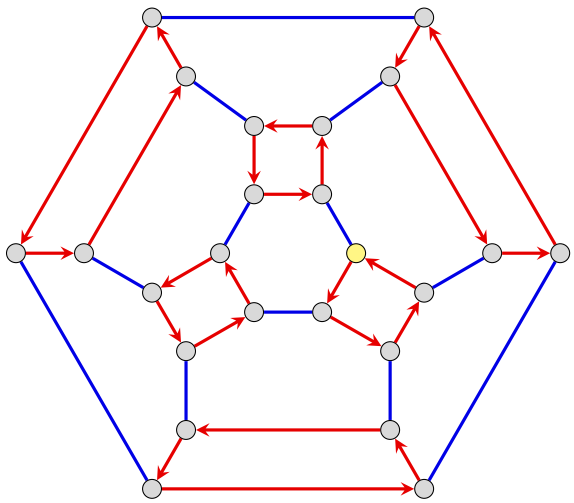
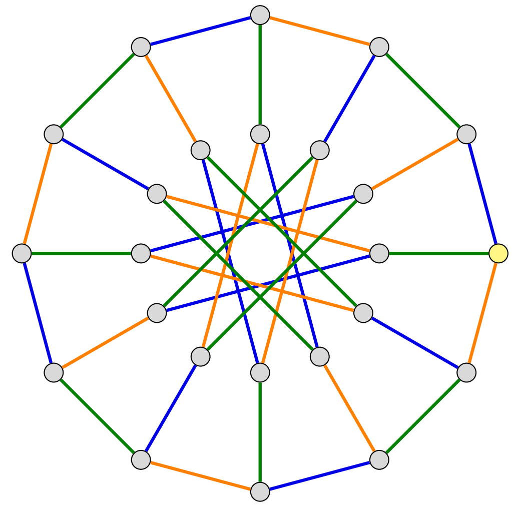
1. Below are two Cayley diagrams for the symmetric group

$$S_4 = \langle (1234), (12) \rangle = \langle (12), (13), (14) \rangle.$$

At left is a truncated octahedron, called the *pemutohedron*. At right is the *Nauru graph*.

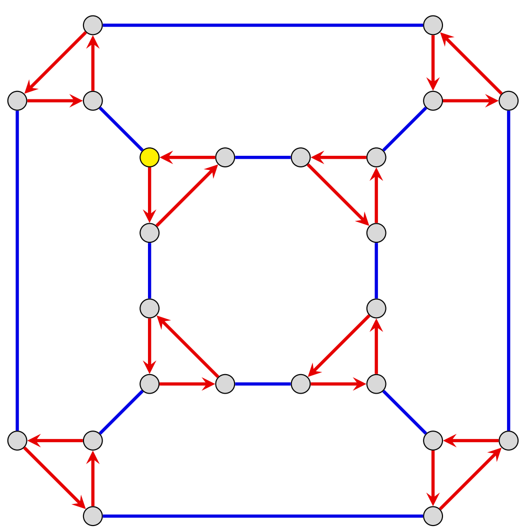
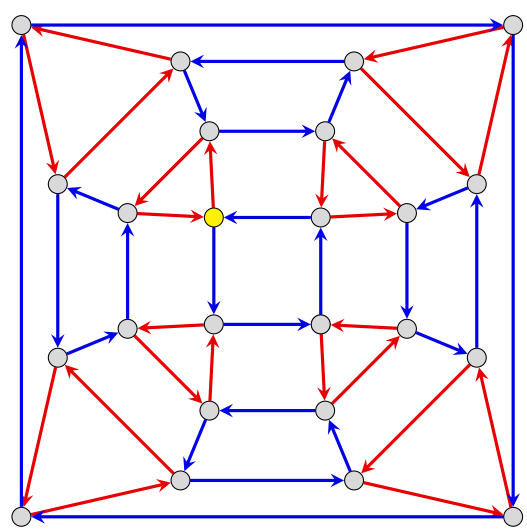
Carry out the following steps, taking the yellow node to represent the identity.

(a) On both diagrams, label the nodes by elements of S_4, written in cycle notation as a product of disjoint cycles.

(b) On the Nauru graph, label the nodes with permutations of the word **1234**, where $(i\ j)$ swaps the i^{th} and j^{th} *coordinates*.

(c) On a separate copy of the Nauru graph, label the nodes with permutations of **1234**, where $(i\ j)$ swaps the *numbers* i and j.

2. Two Cayley diagrams for the symmetric group S_4 arranged on flattened Archimedean solids – the truncated cube (left) and the rhombicuboctahedron (right).

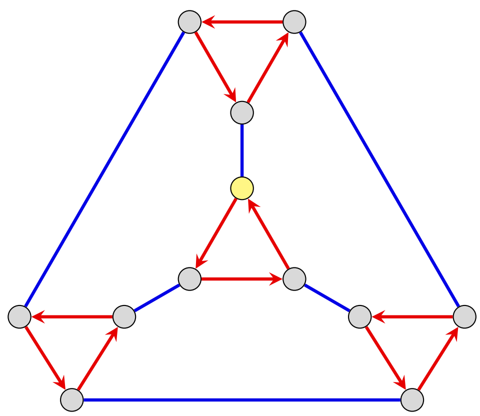



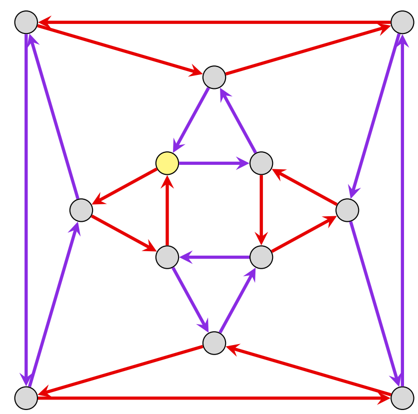
Determine what generating sets will yield these Cayley diagrams. Then, label the nodes with permutations in cycle notation, written as a product of disjoint cycles.

3. The *alternating* group A_4 is the subgroup of S_4 that consists of the even permutations. Two Cayley diagrams are shown below, for presentations

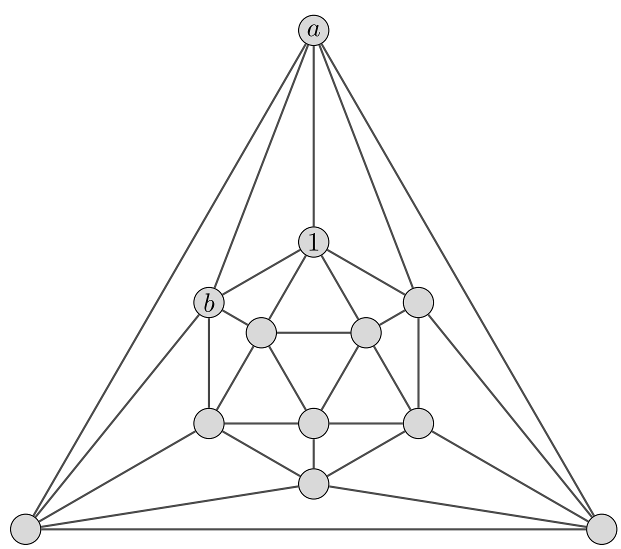
$$A_4 = \big\langle (123), (12)(34) \big\rangle = \big\langle (123), (234) \big\rangle.$$

Label the nodes of these diagrams with elements of A_4 in cycle notation, written as a product of disjoint cycles.





4. Draw the Cayley diagram of the group $G = \langle a, b, c \mid a^2 = b^3 = c^3 = abc = 1 \rangle$ on the skeleton of the icosahedron, shown below, and label the nodes with elements written using a, b, and c.



There are five groups of order 12: the abelian groups C_{12} and $C_6 \times C_2$, the dihedral group D_6, the alternating group A_4, and the dicyclic group Dic_6. Determine which group G is isomorphic to, and then re-draw this Cayley diagram with the nodes labeled with elements of that group.

5. Prove that if $g^2 = e$ for all $g \in G$, then G must be abelian.