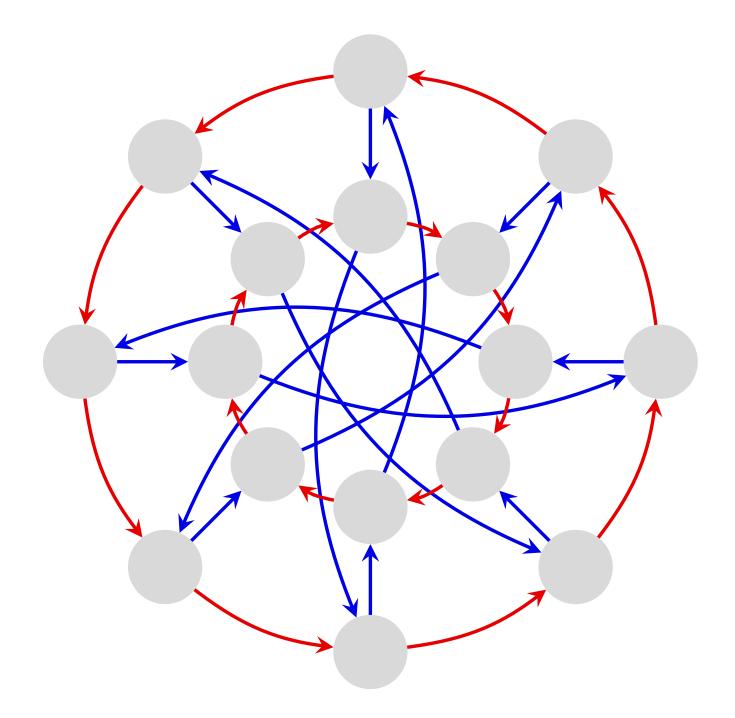
Supplemental material for Visual Algebra (Math 4120), ${ m HW}~2$

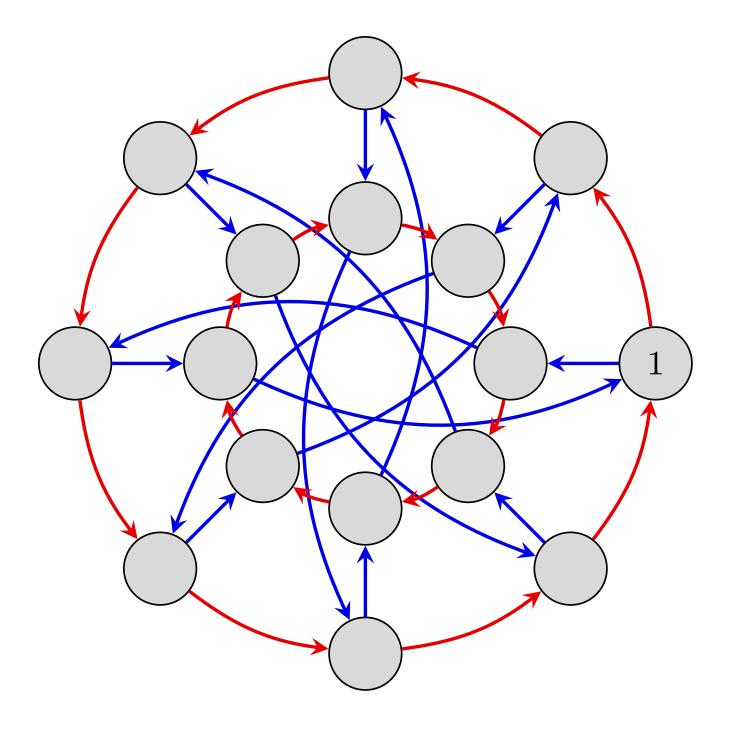
#3(a): Cayley graph for the generalized quaternion group

$$Q_{16} = \left\langle \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i, j \right\rangle,$$

where $\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i = e^{2\pi i/8} = \zeta_8$ is an 8th root of unity.



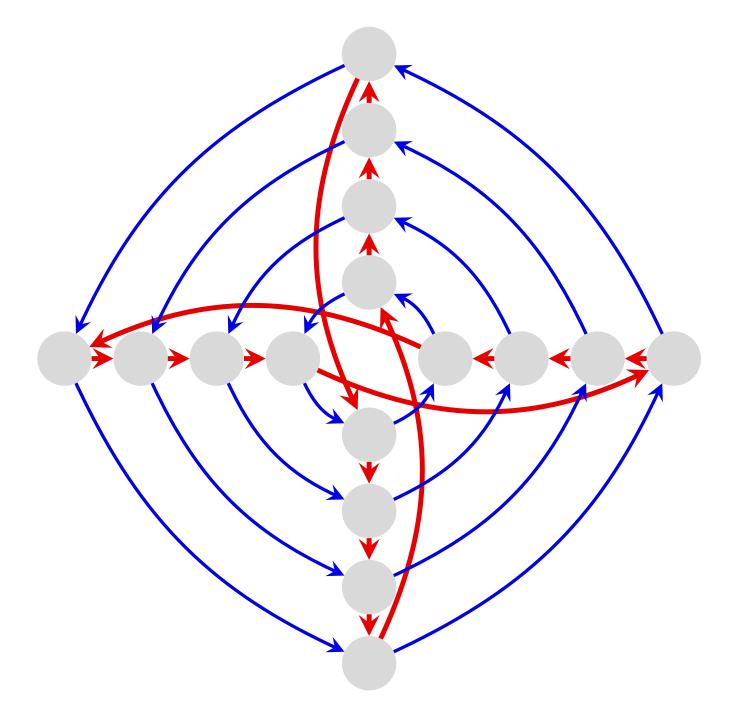
#3(a): Cayley graph for the generalized quaternion group $Q_{16} = \langle \zeta_8, j \rangle$, where $\zeta_8 = e^{2\pi i/8}$ is an 8th root of unity.



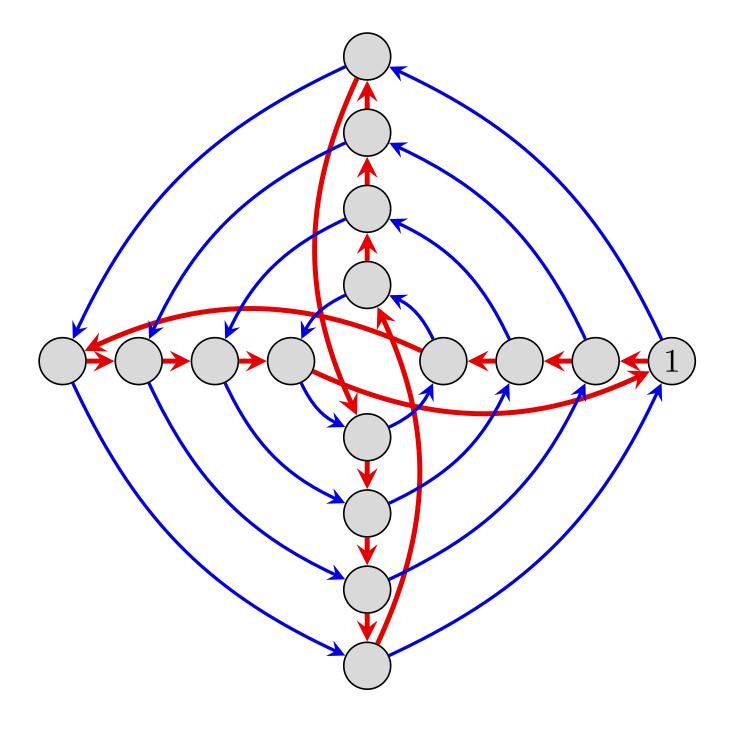
#3(a): Another way to lay out the Cayley graph for the generalized quaternion group

$$Q_{16} = \left\langle \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i, j \right\rangle,$$

where $\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i = e^{2\pi i/8} = \zeta_8$ is an 8th root of unity.



#3(a): Another way to lay out the Cayley graph for the generalized quaternion group $Q_{16} = \langle \zeta_8, j \rangle$, where $\zeta_8 = e^{2\pi i/8}$ is an 8th root of unity.



#3(b): Cayley table of a quotient of the generalized quaternion group

 $Q_{16} = \left\langle \zeta_8, j \right\rangle, \quad \text{where} \quad \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i = e^{2\pi i/8} = \zeta_8,$ by the subgroup $\left\langle \zeta^4 \right\rangle = \left\langle -1 \right\rangle = \{1, -1\}.$

| | ±1 | $\pm \zeta$ | $\pm \zeta^2$ | $\pm \zeta^3$ | $\pm j$ | $\pm \zeta j$ | $\pm \zeta^2 j$ | $\pm \zeta^3 j$ |
|-----------------|----|-------------|---------------|---------------|---------|---------------|-----------------|-----------------|
| ±1 | | | | | | | | |
| $\pm\zeta$ | | | | | | | | |
| $\pm\zeta^2$ | | | | | | | | |
| $\pm \zeta^3$ | | | | | | | | |
| $\pm j$ | | | | | | | | |
| $\pm \zeta j$ | | | | | | | | |
| $\pm \zeta^2 j$ | | | | | | | | |
| $\pm \zeta^3 j$ | | | | | | | | |