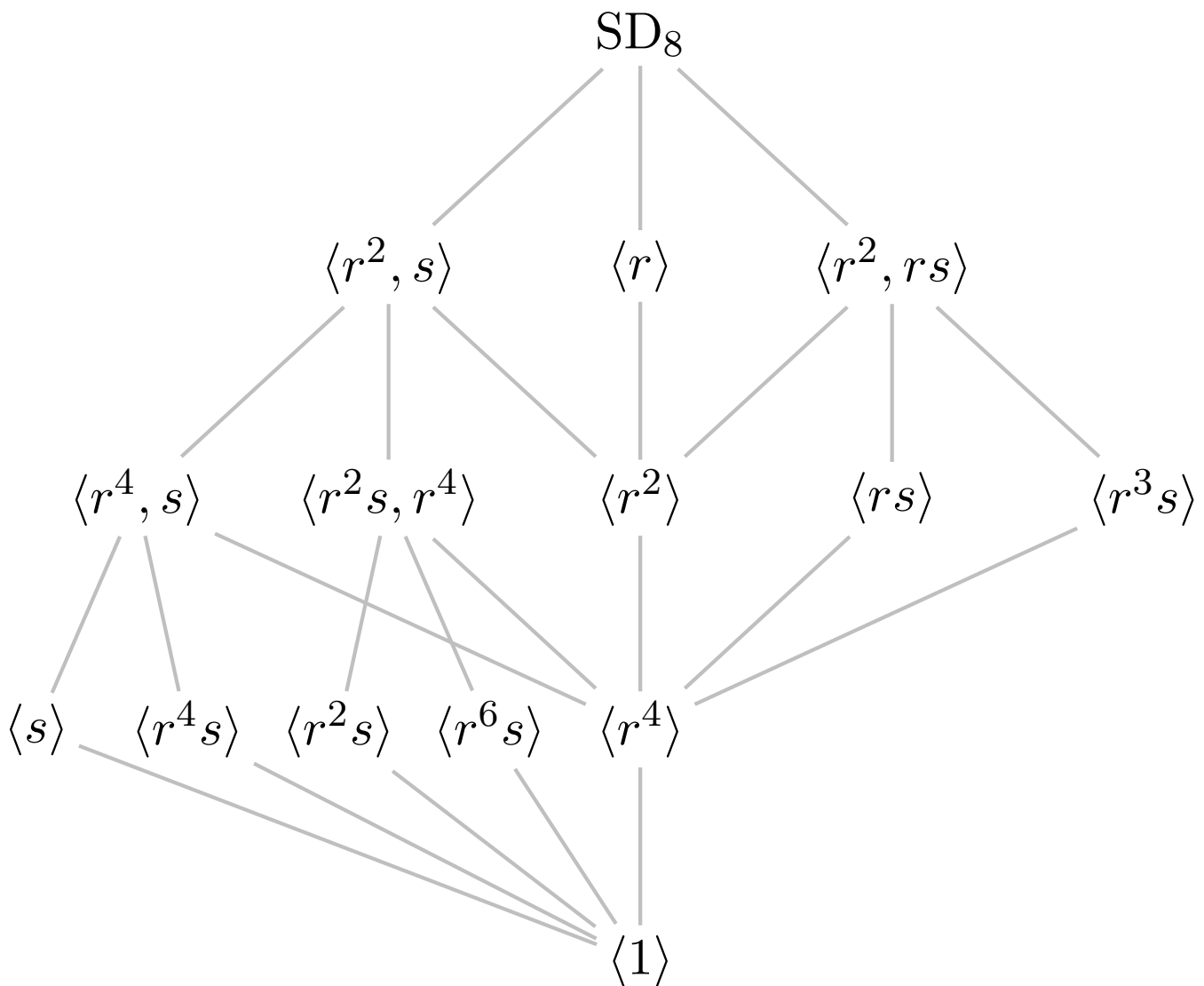
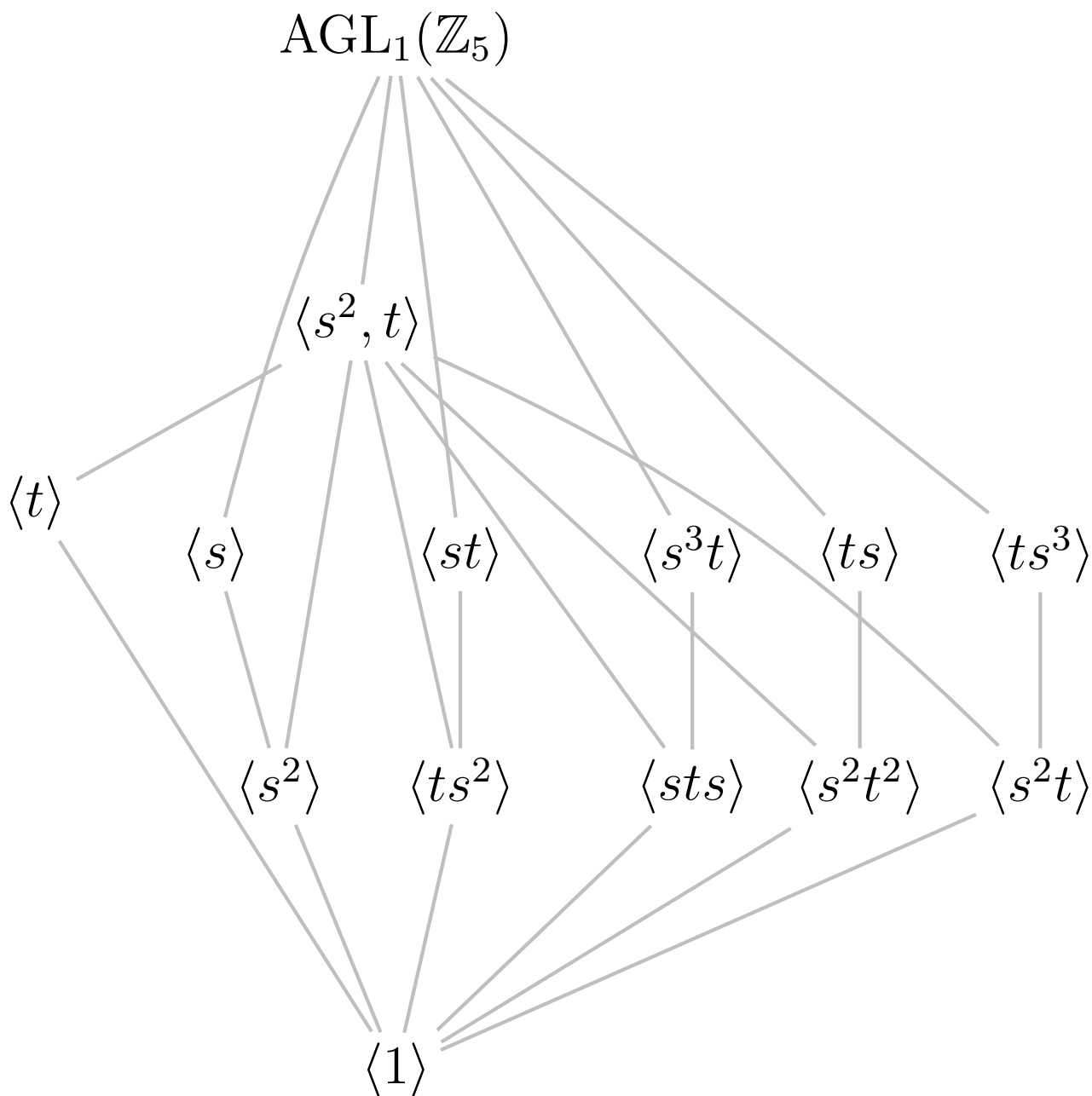


## Supplemental material for Visual Algebra (Math 4120), HW 9

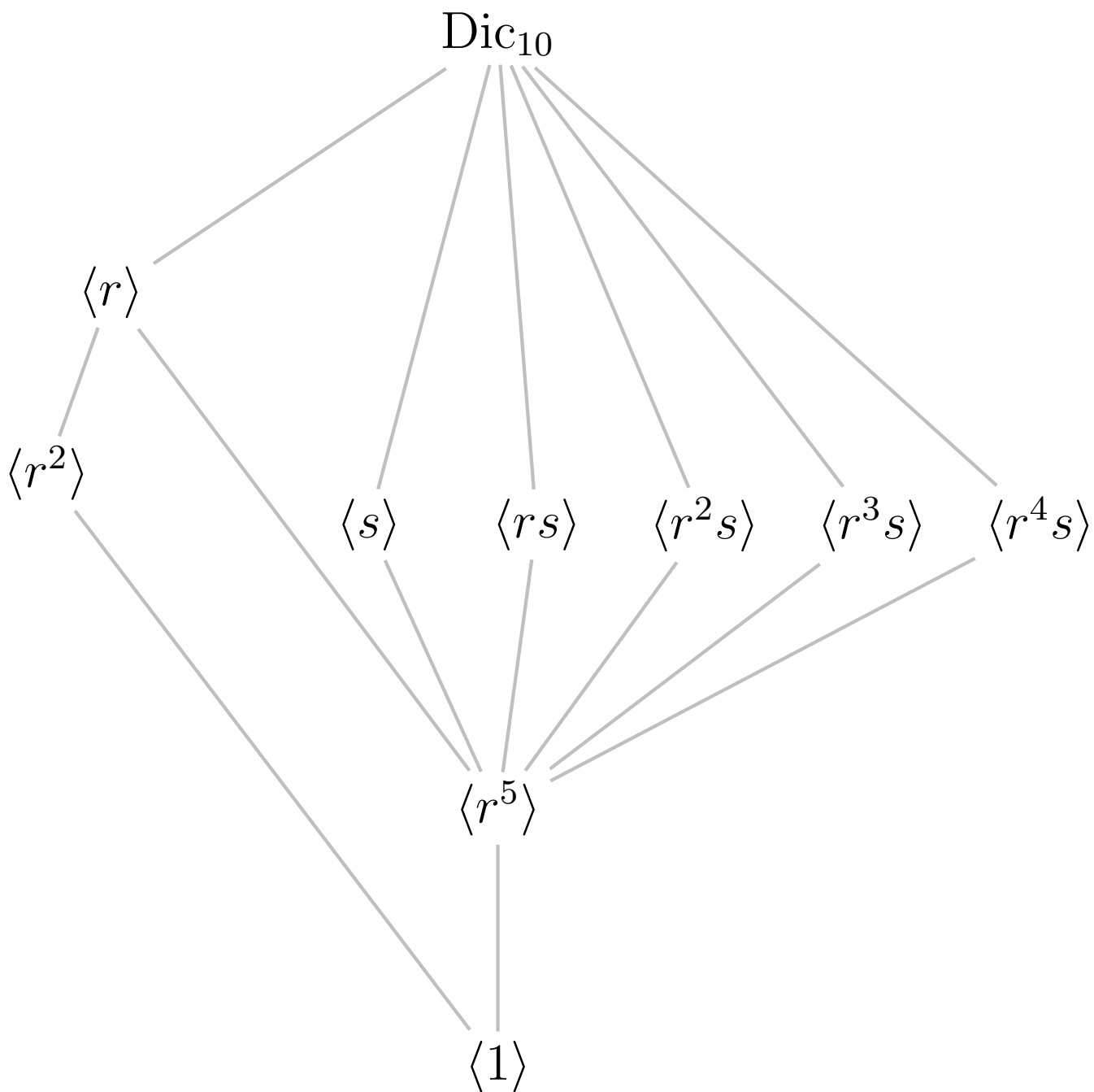
**#2(c)**: Subgroup lattice of the *semidihedral group*  $SD_8$ , grouped by conjugacy classes, with the  $k^{\text{th}}$  commutator subgroups  $G^{(k)}$  included, and sublattice of the quotients  $G^{(k)}/G^{(k-1)}$  identified, along with its isomorphism type.



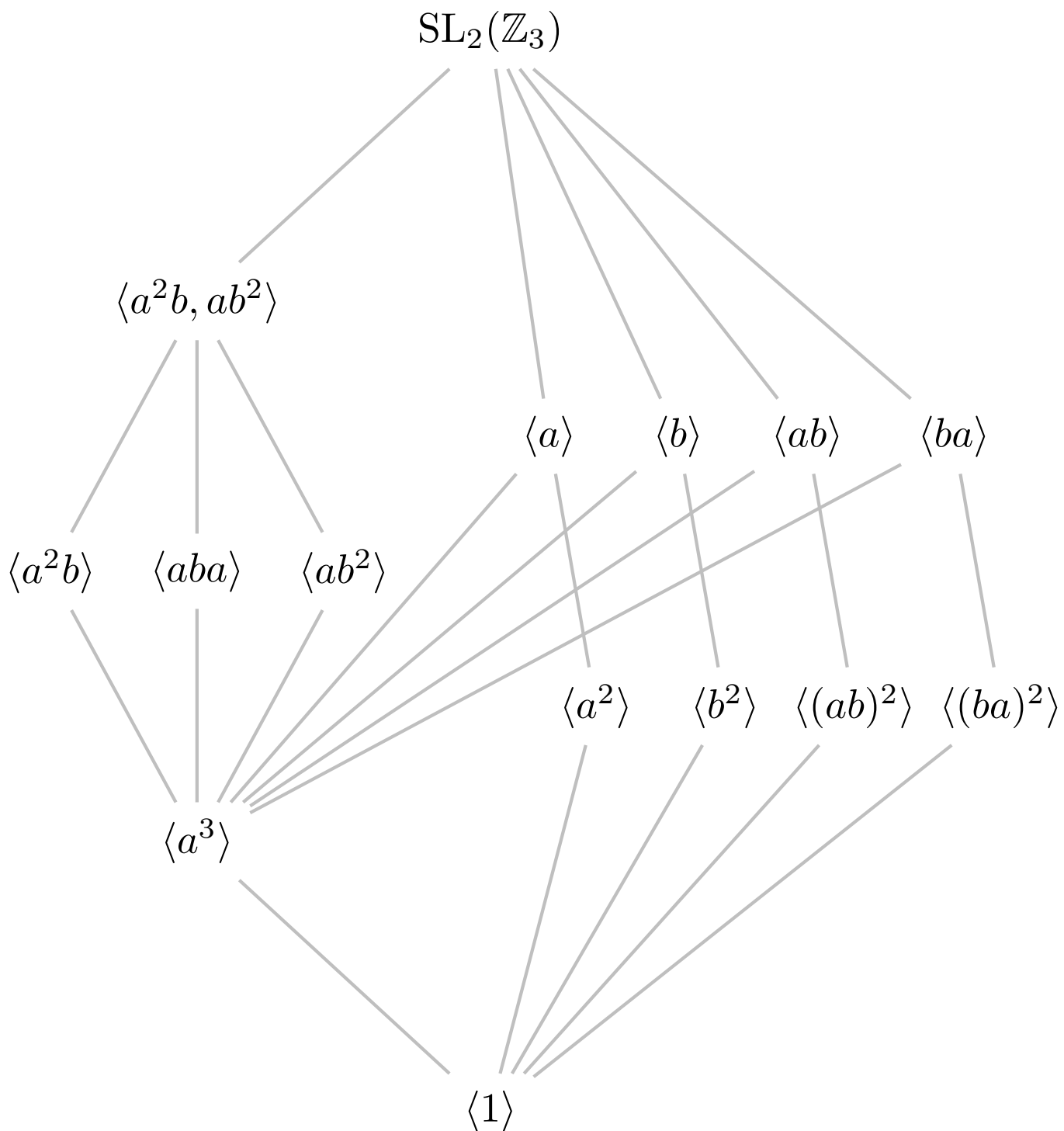
**#2(c)**: Subgroup lattice of the *affine general linear group*  $G = \text{AGL}_1(\mathbb{Z}_5)$ , grouped by conjugacy classes, with the  $k^{\text{th}}$  commutator subgroups  $G^{(k)}$  included, and sublattice of the quotients  $G^{(k)}/G^{(k-1)}$  identified, along with its isomorphism type.



**#2(c)**: Subgroup lattice of the *dicyclic group*  $\text{Dic}_{10}$ , grouped by conjugacy classes, with the  $k^{\text{th}}$  commutator subgroups  $G^{(k)}$  included, and sublattice of the quotients  $G^{(k)}/G^{(k-1)}$  identified, along with its isomorphism type.

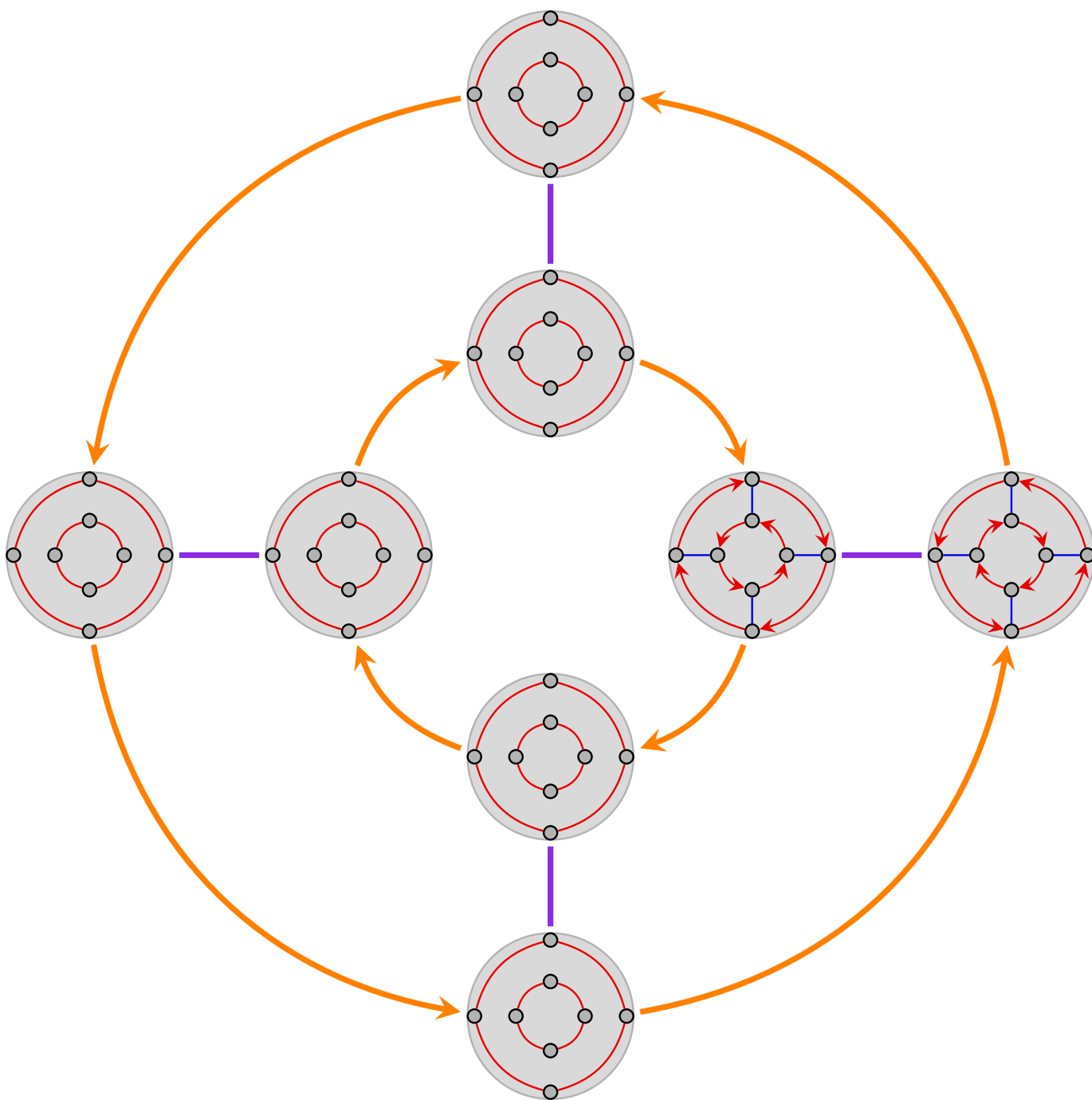


**#2(c)**: Subgroup lattice of the *special linear group*  $G = \mathrm{SL}_2(\mathbb{Z}_3)$ , grouped by conjugacy classes, with the  $k^{\mathrm{th}}$  commutator subgroups  $G^{(k)}$  included, and sublattice of the quotients  $G^{(k)}/G^{(k-1)}$  identified, along with its isomorphism type.



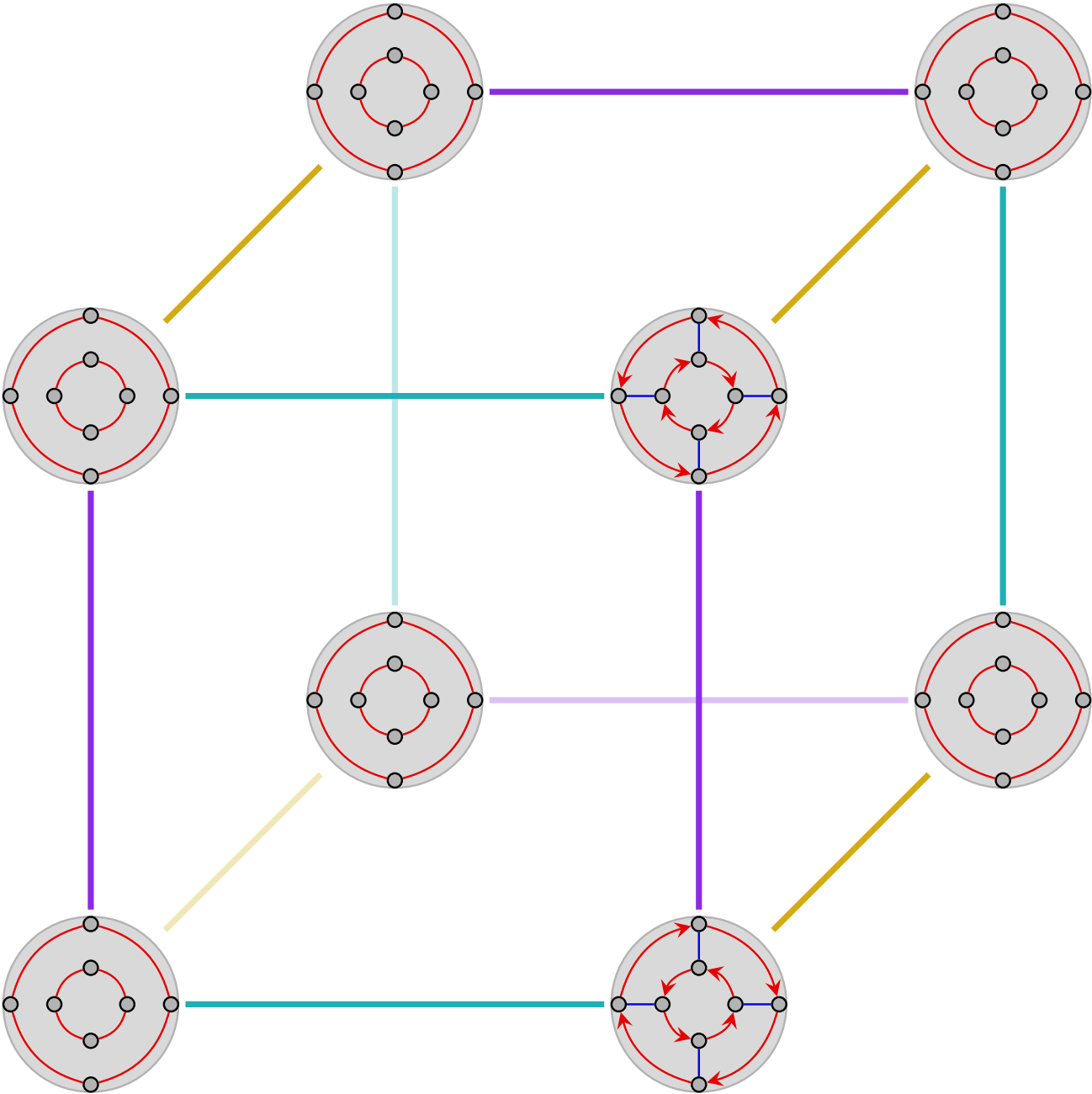
**#3(a,c):** Cayley diagram of the automorphism group  $\text{Aut}(D_4) \cong D_4$ , with the nodes labeled by re-wired copies of the Cayley diagram of  $D_4 = \langle r, f \rangle$ , and also denoted with the corresponding element from

$$\text{Aut}(D_4) = \{ \text{Id}, \varphi_r, \varphi_f, \varphi_{rf}, \omega, \varphi_r\omega, \varphi_f\omega, \varphi_{rf}\omega \} = \text{Inn}(D_4) \cup \text{Inn}(D_4)\omega.$$



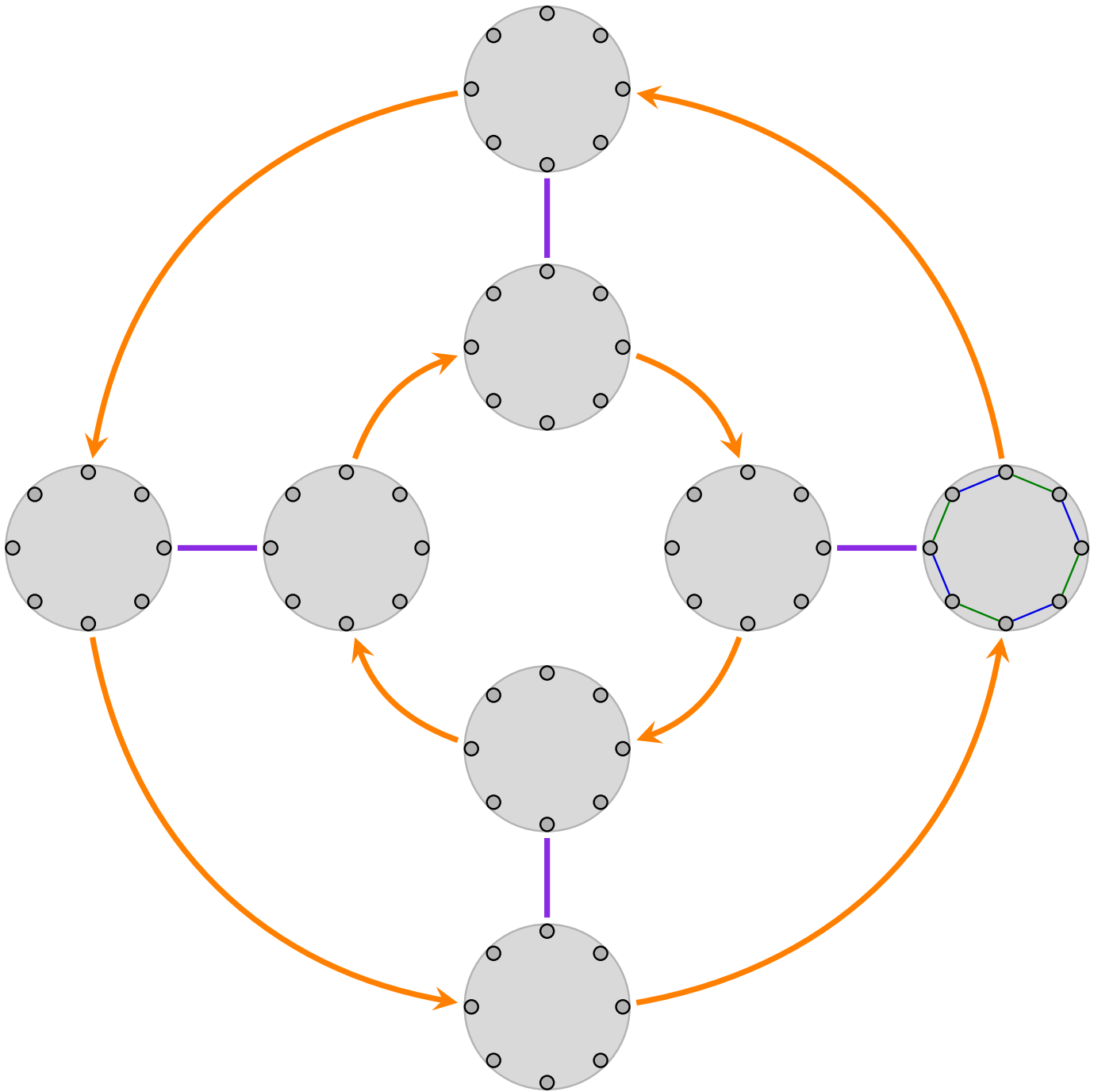
**#3(a,c):** Cayley diagram of the automorphism group  $\text{Aut}(D_4) \cong V_4 \times C_2 \cong D_4$ , with the nodes labeled by re-wired copies of the Cayley diagram of  $D_4 = \langle r, f \rangle$ , and also denoted with the corresponding element from

$$\text{Aut}(D_4) = \{ \text{Id}, \varphi_r, \varphi_f, \varphi_{rf}, \omega, \varphi_r\omega, \varphi_f\omega, \varphi_{rf}\omega \} = \text{Inn}(D_4) \cup \text{Inn}(D_4)\omega.$$



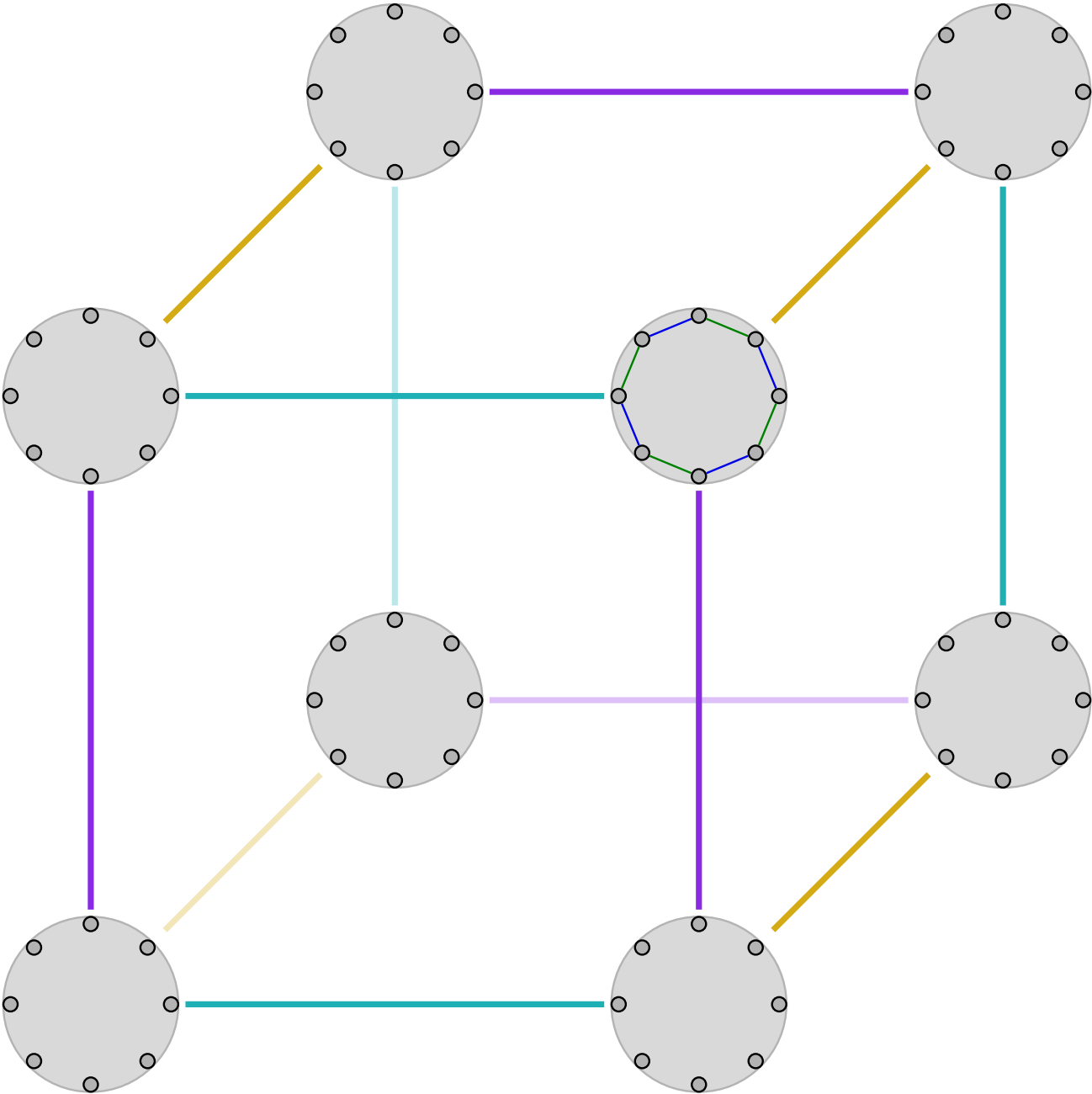
**#3(b,c):** Cayley diagram of the automorphism group  $\text{Aut}(D_4) \cong V_4 \rtimes C_2 \cong D_4$ , with the nodes labeled by re-wired copies of the Cayley diagram of  $D_4 = \langle r, f \rangle$ , and also denoted with the corresponding element from

$$\text{Aut}(D_4) = \{ \text{Id}, \varphi_r, \varphi_f, \varphi_{rf}, \omega, \varphi_r\omega, \varphi_f\omega, \varphi_{rf}\omega \} = \text{Inn}(D_4) \cup \text{Inn}(D_4)\omega.$$



**#3(b,c):** Cayley diagram of the automorphism group  $\text{Aut}(D_4) \cong V_4 \rtimes C_2 \cong D_4$ , with the nodes labeled by re-wired copies of the Cayley diagram of  $D_4 = \langle s, t \rangle$ , and also denoted with the corresponding element from

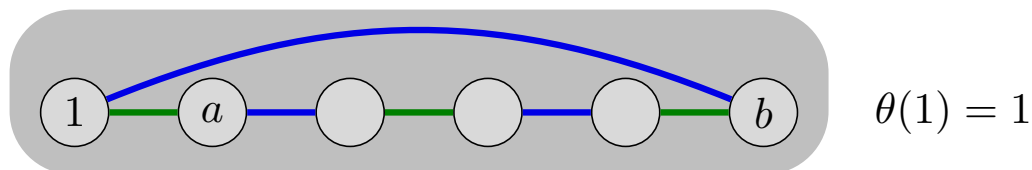
$$\text{Aut}(D_4) = \{ \text{Id}, \varphi_r, \varphi_f, \varphi_{rf}, \omega, \varphi_r\omega, \varphi_f\omega, \varphi_{rf}\omega \} = \text{Inn}(D_4) \cup \text{Inn}(D_4)\omega.$$





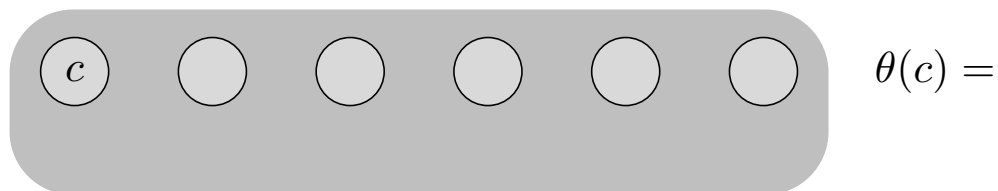
#4(i–ii): Both  $D_3 \times C_2$  and  $D_3 \times C_2$  are semidirect products, and each is defined by a “labeling map”

$$\theta: C_2 \longrightarrow \text{Aut}(D_3) = \langle \alpha, \beta \mid \alpha^3 = \beta^2 = (\alpha\beta)^2 = 1 \rangle \cong D_3.$$

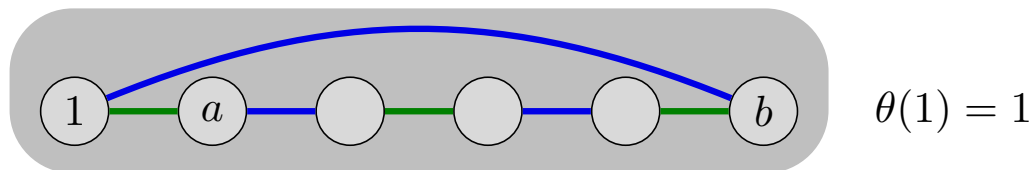


$$\theta(1) = 1$$

$D_3 \times C_2$

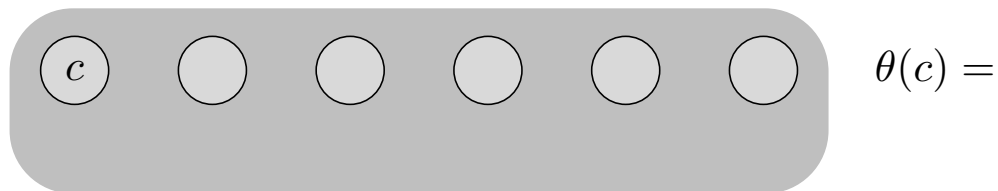


$$\theta(c) =$$



$$\theta(1) = 1$$

$D_3 \times C_2$



$$\theta(c) =$$

#4(iii–iv): The semidirect product  $A \rtimes B$  is defined by “labeling map”  $\theta: B \rightarrow \text{Aut}(A)$ . Here are  $V_4 \rtimes C_3$  and  $C_3 \rtimes V_4$  and  $\text{Aut}(V_4) = \langle \alpha, \beta \rangle \cong D_3$  and  $\text{Aut}(C_3) = \langle 1, \phi \rangle \cong C_2$ .

