Scratch paper for Abstract Algebra (Math 8510), HW 4
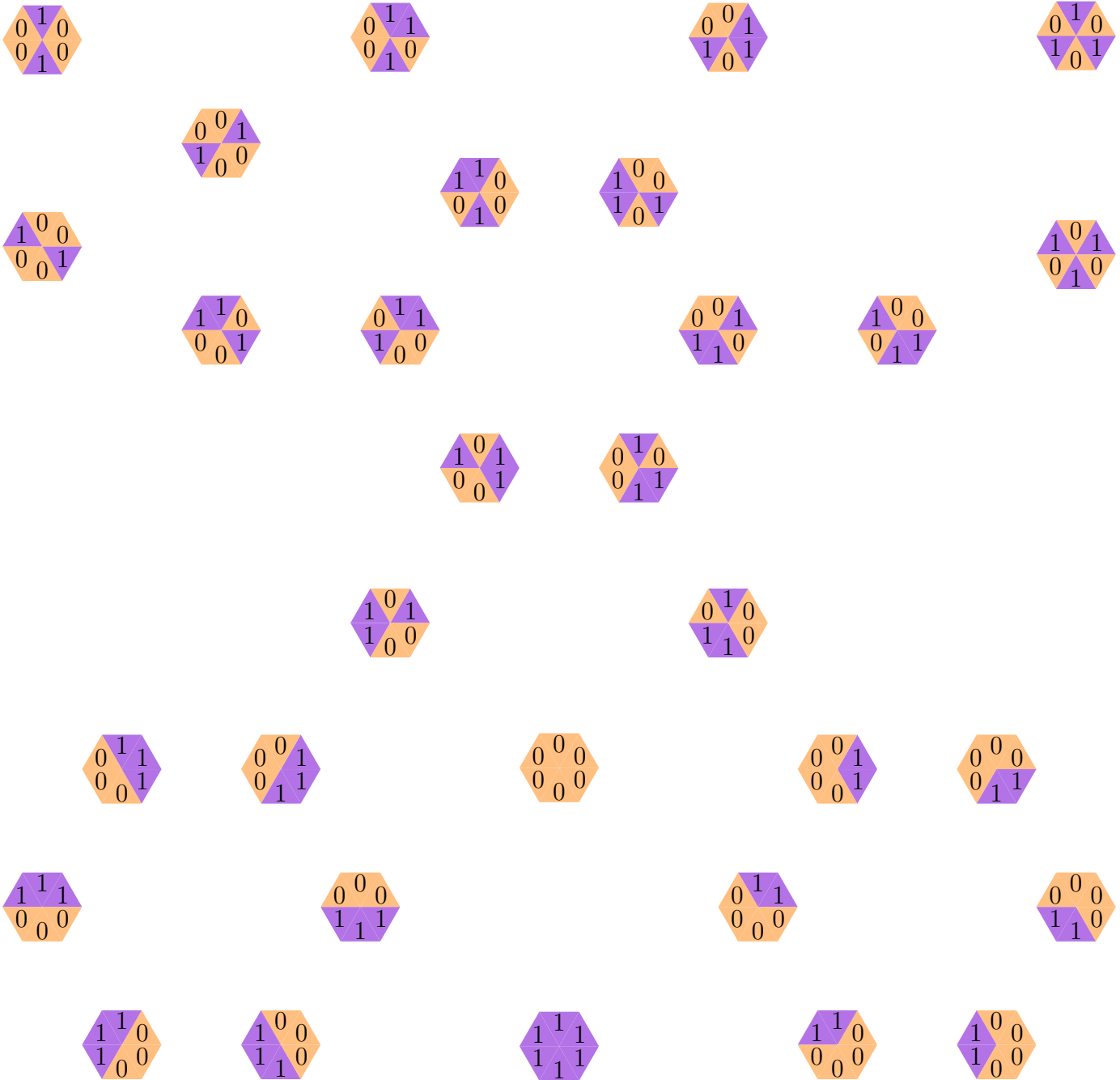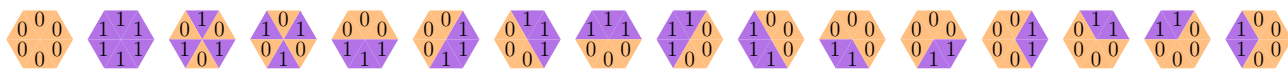
#1(a): Action graph of $D_6 = \langle r, f \rangle$ acting on 31 "binary hexagons."
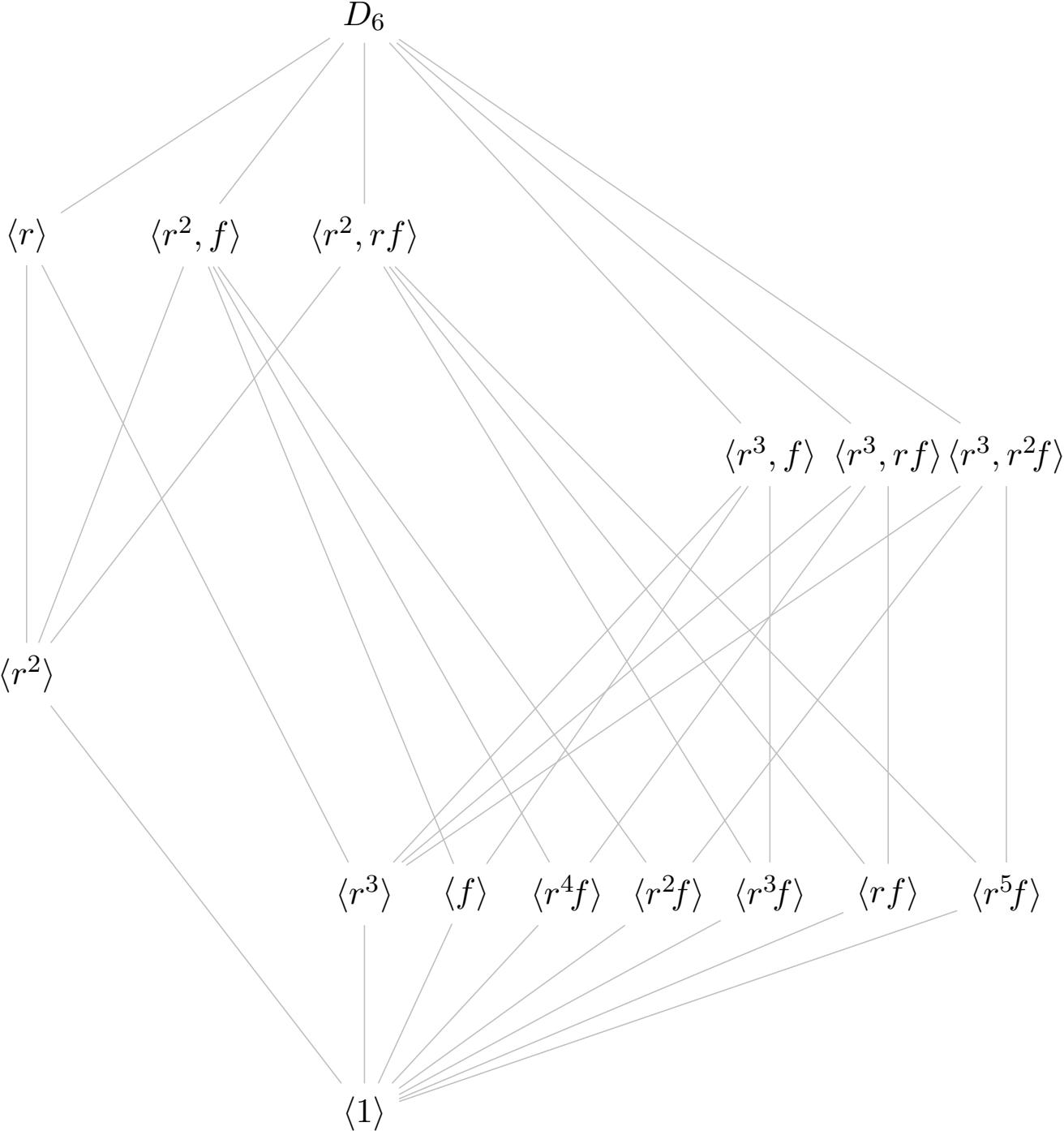
$\#\mathbf{1(a)}$: Fixed point table of D_6 acting on "binary hexagons."

1																
r																
r^2																
r^3																
r^4																
r^5																
f																
rf																
r^2f																
r^3f																
r^4f																
r^5f																

#1(a): Fixed point table of D_6 acting on "binary hexagons" (contin.)

1														
r														
r^2														
r^3														
r^4														
r^5														
f														
rf														
r^2f														
r^3f														
r^4f														
r^5f														

#1(b): Action graph of $D_6 = \langle r, f \rangle$ acting on its subgroups by conjugation.

D_6

$\langle r \rangle$ $\langle r^2, f \rangle$ $\langle r^2, rf \rangle$

$\langle r^3, f \rangle$ $\langle r^3, rf \rangle$ $\langle r^3, r^2f \rangle$

$\langle r^2 \rangle$

$\langle r^3 \rangle$ $\langle f \rangle$ $\langle r^4f \rangle$ $\langle r^2f \rangle$ $\langle r^3f \rangle$ $\langle rf \rangle$ $\langle r^5f \rangle$

$\langle 1 \rangle$

#1(b): Fixed point table of D_6 acting on its subgroups by conjugation.

|	D_6	$\langle r \rangle$	$\langle r^2, f \rangle$	$\langle r^2, rf \rangle$	$\langle r^3, f \rangle$	$\langle r^3, rf \rangle$	$\langle r^3, r^2f \rangle$	$\langle r^2 \rangle$	$\langle r^3 \rangle$	$\langle f \rangle$	$\langle r^2f \rangle$	$\langle r^4f \rangle$	$\langle rf \rangle$	$\langle r^3f \rangle$	$\langle r^5f \rangle$
1															
r															
r^2															
r^3															
r^4															
r^5															
f															
rf															
r^2f															
r^3f															
r^4f															
r^5f															

#1(c): The action graph poset of $D_6 = \langle r, f \rangle$, constructed by collapsing its Cayley graph by right cosets of its subgroups.
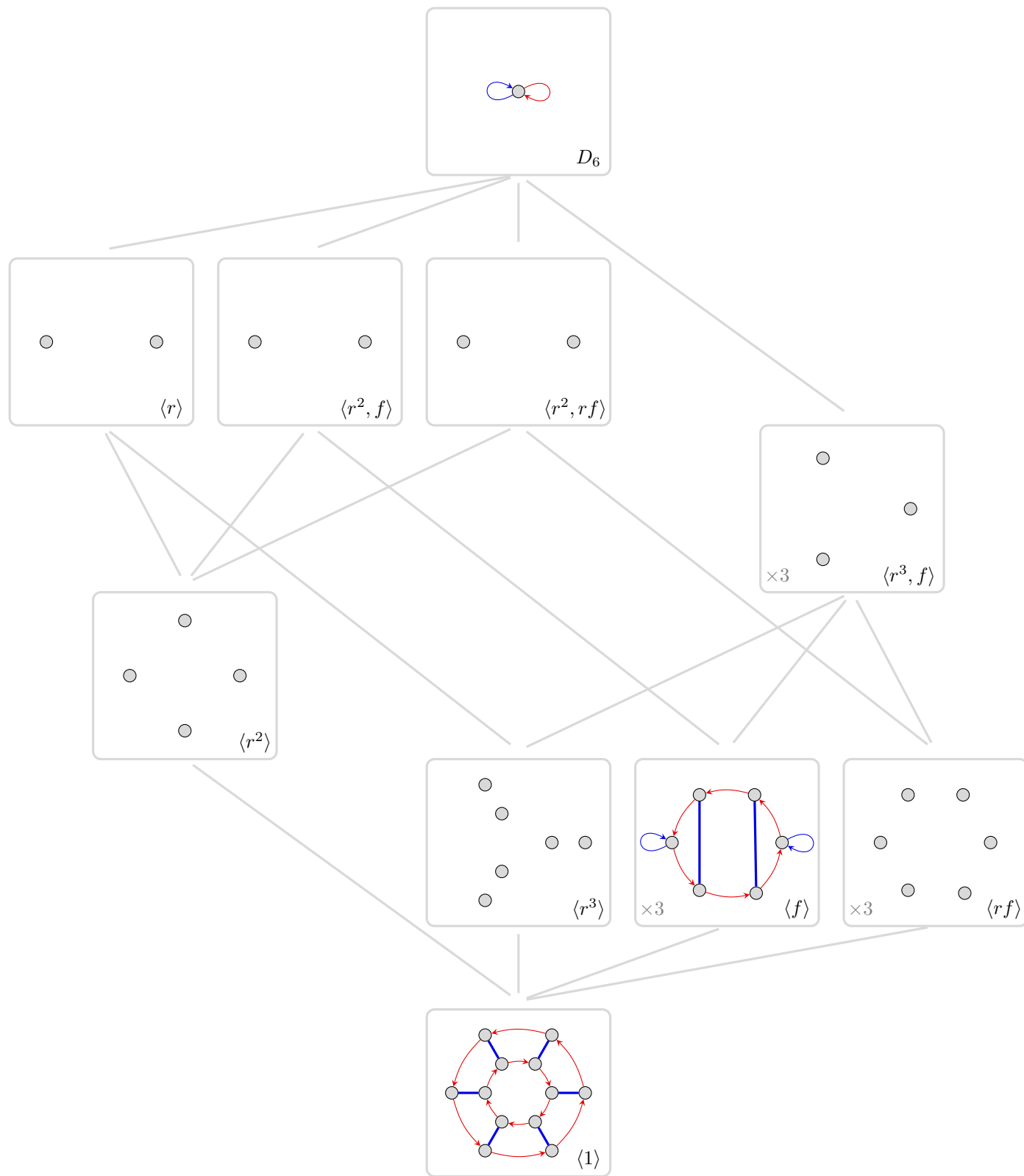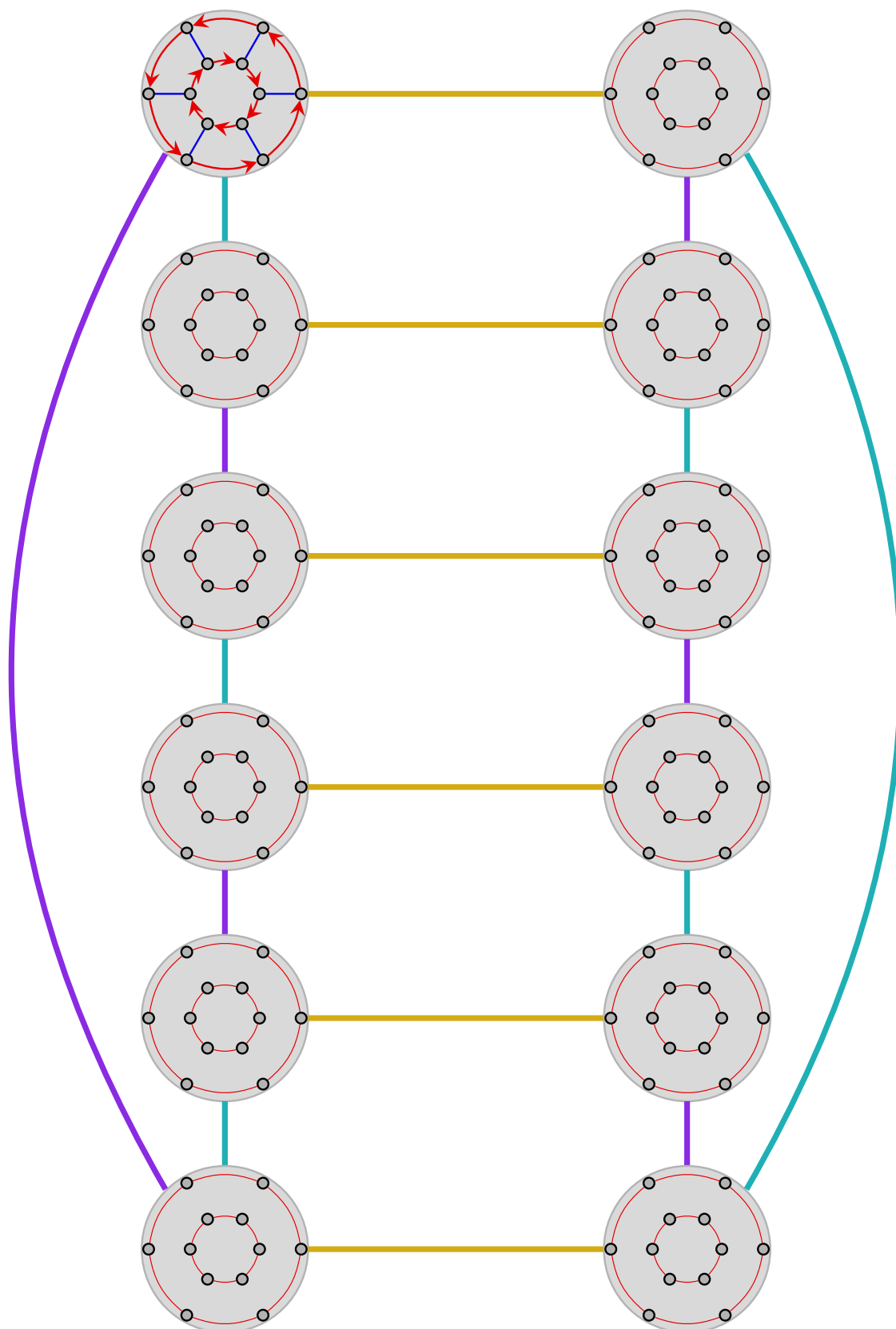
#1(d): Cayley graph of the automorphism group $\mathrm{Aut}(D_6) \cong D_6$, with the nodes labeled by rewired copies of the Cayley graph of $D_6 = \langle \textcolor{red}{r}, \textcolor{blue}{f} \rangle$, and with the inner automorphisms labeled.
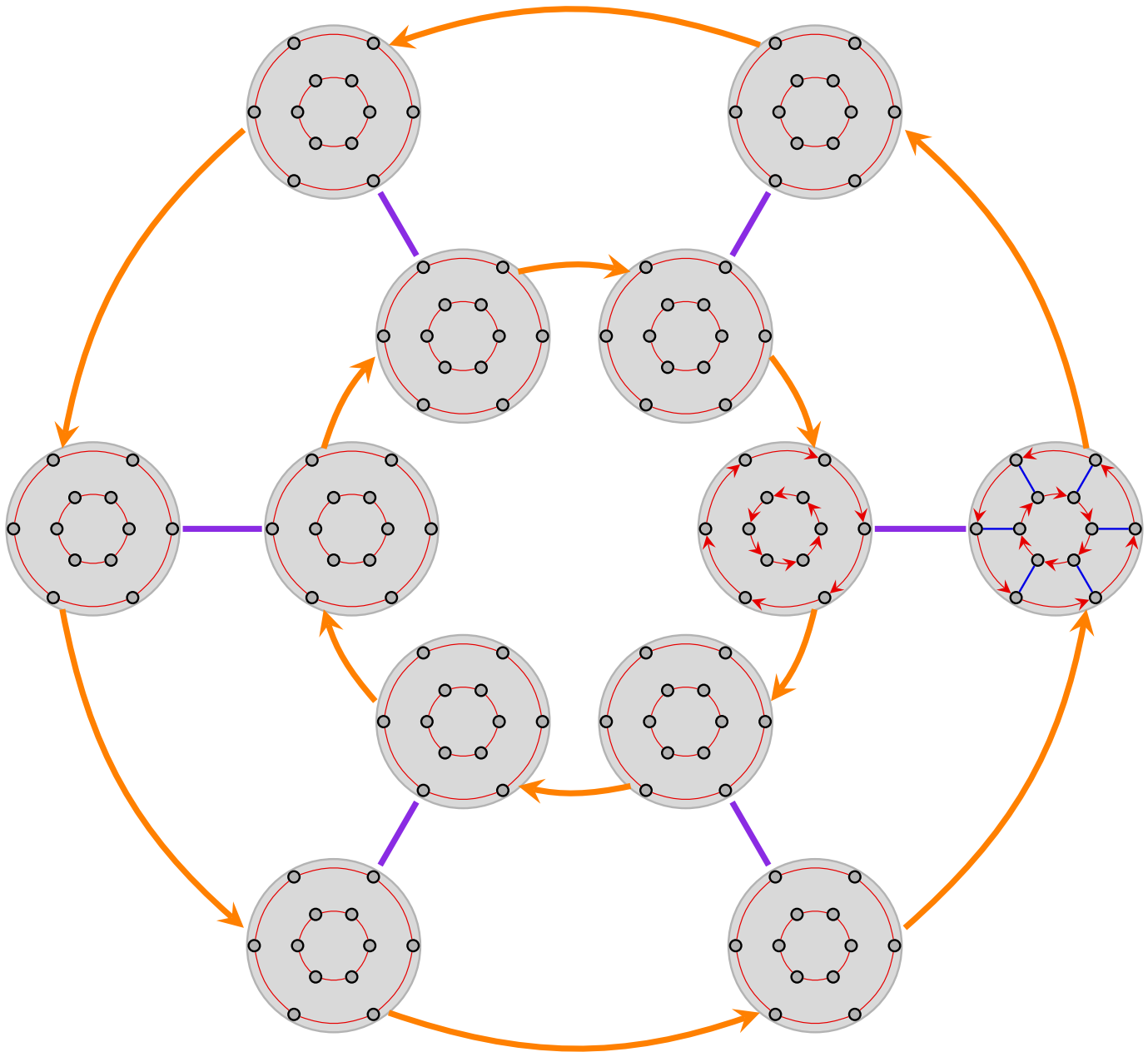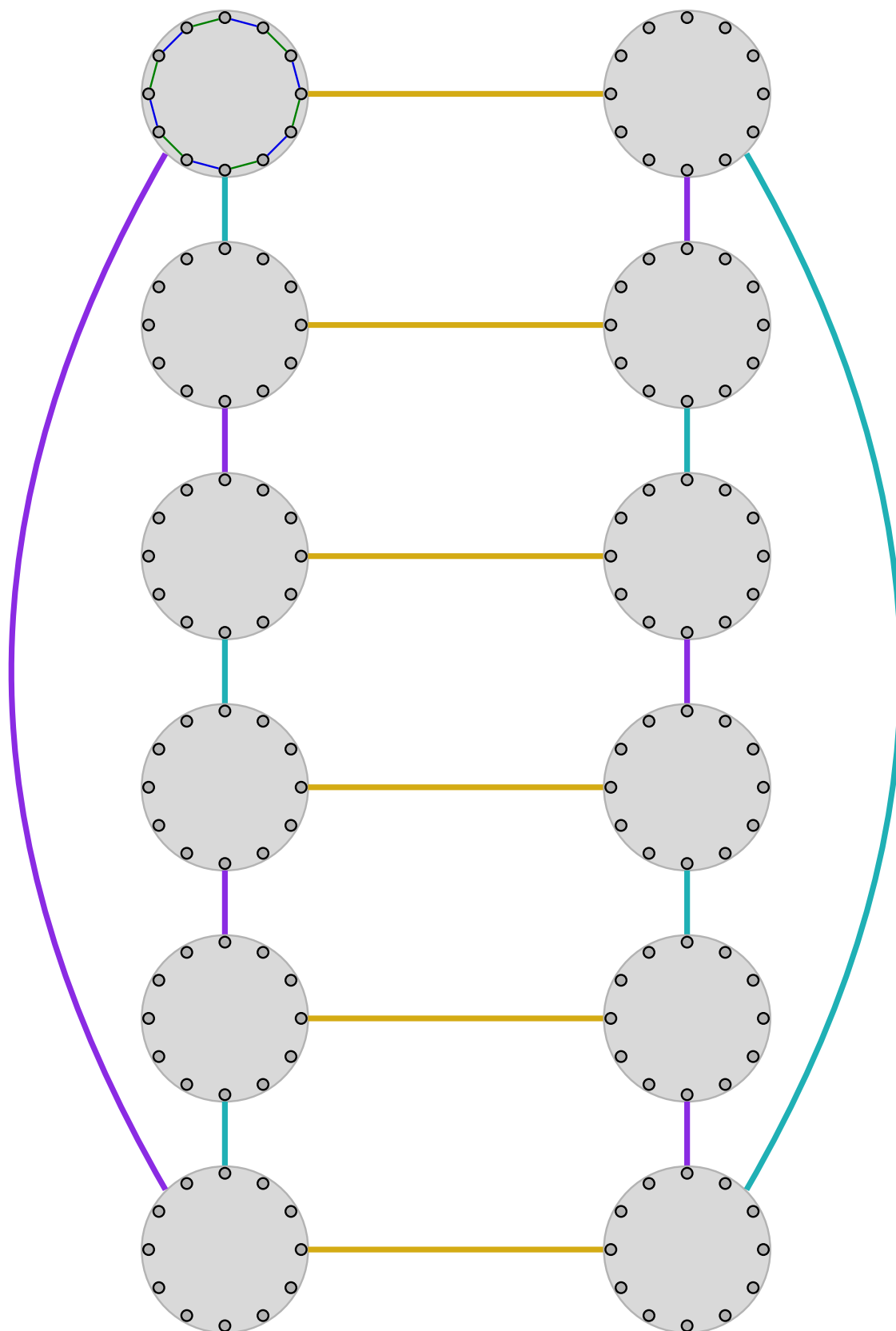
#1(d): Cayley graph of the automorphism group $\mathrm{Aut}(D_6) \cong D_6$, with the nodes labeled by rewired copies of the Cayley graph of $D_6 = \langle s, t \rangle = \langle f, rf \rangle$, and with the inner automorphisms labeled.
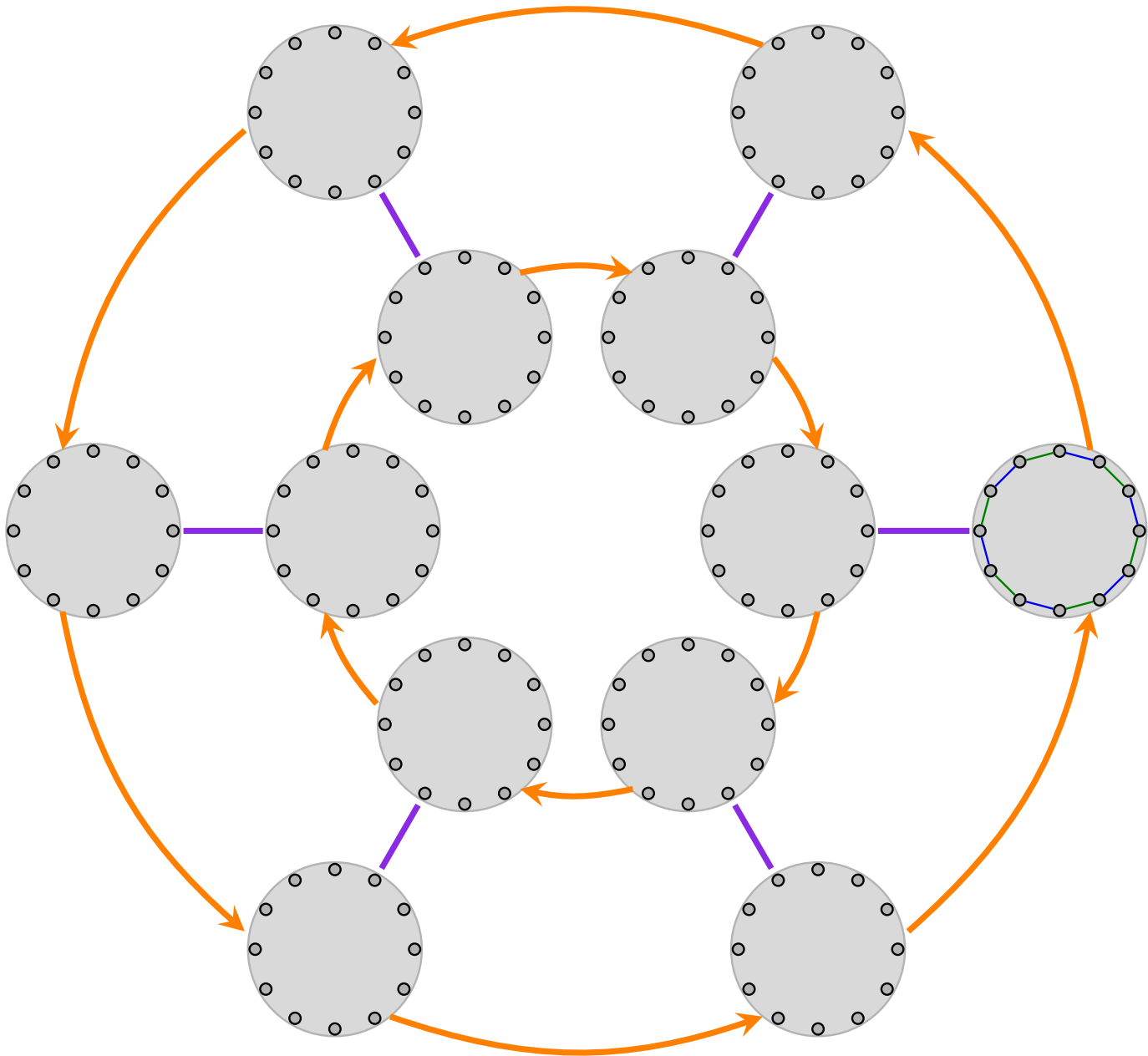
#1(d): Cayley graph of the automorphism group $\mathrm{Aut}(D_6) \cong D_3 \rtimes C_2$.

#1(d): Cayley graph of the automorphism group $\mathrm{Aut}(D_6) \cong D_3 \rtimes C_2$.

#1(e): Partition of $\mathrm{Aut}(D_6) \cong \mathrm{Inn}(D_6) \rtimes \langle \eta \rangle$ into cosets of $\mathrm{Inn}(D_6)$.

$\mathrm{Inn}(D_6) = \langle \varphi_r, \varphi_f \rangle$ | | | | $\mathrm{Inn}(D_6)\omega$

Id

1	r	r^2	f	r^2f	r^4f
r^3	r^5	r^4	rf	r^3f	r^5f

ω

1	r	r^2	f	r^2f	r^4f
r^3	r^5	r^4	rf	r^3f	r^5f

φ_r

1	r	r^2	f	r^2f	r^4f
r^3	r^5	r^4	rf	r^3f	r^5f

$\varphi_r\omega$

1	r	r^2	f	r^2f	r^4f
r^3	r^5	r^4	rf	r^3f	r^5f

φ_{r^2}

1	r	r^2	f	r^2f	r^4f
r^3	r^5	r^4	rf	r^3f	r^5f

$\varphi_{r^2}\omega$

1	r	r^2	f	r^2f	r^4f
r^3	r^5	r^4	rf	r^3f	r^5f

φ_f

1	r	r^2	f	r^2f	r^4f
r^3	r^5	r^4	rf	r^3f	r^5f

$\varphi_f\omega$

1	r	r^2	f	r^2f	r^4f
r^3	r^5	r^4	rf	r^3f	r^5f

φ_{rf}

1	r	r^2	f	r^2f	r^4f
r^3	r^5	r^4	rf	r^3f	r^5f

$\varphi_{rf}\omega$

1	r	r^2	f	r^2f	r^4f
r^3	r^5	r^4	rf	r^3f	r^5f

φ_{r^2f}

1	r	r^2	f	r^2f	r^4f
r^3	r^5	r^4	rf	r^3f	r^5f

$\varphi_{r^2f}\omega$

1	r	r^2	f	r^2f	r^4f
r^3	r^5	r^4	rf	r^3f	r^5f

#1(f): Subgroup lattice of $\mathrm{Aut}(D_6) \cong \mathrm{Inn}(D_4) \rtimes \langle \eta \rangle = \langle \varphi_r, \varphi_f, \eta \rangle$.

$\mathrm{Aut}(D_6)$

#1(h): Action graph and fixed point table of the action of $\mathrm{Aut}(D_6) = \langle \varphi_r, \varphi_f, \eta \rangle \cong \mathrm{Inn}(D_6) \rtimes \langle \eta \rangle$ on D_6.

1	r	r^2	f	$r^2 f$	$r^4 f$
r^3	r^5	r^4	rf	$r^3 f$	$r^5 f$

	1	r	r^2	r^3	r^4	r^5	f	rf	$r^2 f$	$r^3 f$	$r^4 f$	$r^5 f$
Id												
φ_r												
φ_{r^2}												
φ_f												
φ_{rf}												
$\varphi_{r^2 f}$												
η												
$\varphi_r \eta$												
$\varphi_{r^2}\eta$												
$\varphi_f \eta$												
$\varphi_{rf}\eta$												
$\varphi_{r^2 f}\eta$												

#1(h): Action graph and fixed point table of the action of $\mathrm{Aut}(D_6) = \langle \varphi_r, \varphi_f, \omega \rangle$ on D_6, where $\omega \in \mathrm{Aut}(D_6)$ is the outer automorphism $f \mapsto rf$ of order 6, that cyclically rotates the axes of reflection.

1	r	r^2	f	r^2f	r^4f
r^3	r^5	r^4	rf	r^3f	r^5f

	1	r	r^2	r^3	r^4	r^5	f	rf	r^2f	r^3f	r^4f	r^5f
Id												
φ_r												
φ_{r^2}												
φ_f												
φ_{rf}												
φ_{r^2f}												
ω												
$\varphi_r\omega$												
$\varphi_{r^2}\omega$												
$\varphi_f\omega$												
$\varphi_{rf}\omega$												
$\varphi_{r^2f}\omega$												

#1(i): Action graph and fixed point table of the action of $\mathrm{Aut}(D_6) = \langle \varphi_r, \varphi_f, \eta \rangle \cong \mathrm{Inn}(D_6) \rtimes \langle \eta \rangle$ on the conjugacy classes of D_6.

$\mathrm{cl}(1)$ $\mathrm{cl}(r^3)$ $\mathrm{cl}(r)$ $\mathrm{cl}(r^2)$ $\mathrm{cl}(f)$ $\mathrm{cl}(rf)$

	$\mathrm{cl}(1)$	$\mathrm{cl}(r^3)$	$\mathrm{cl}(r)$	$\mathrm{cl}(r^2)$	$\mathrm{cl}(f)$	$\mathrm{cl}(rf)$
Id						
φ_r						
φ_{r^2}						
φ_f						
φ_{rf}						
φ_{r^2f}						
η						
$\varphi_r \eta$						
$\varphi_{r^2} \eta$						
$\varphi_f \eta$						
$\varphi_{rf} \eta$						
$\varphi_{r^2f} \eta$						

#1(i): Action graph and fixed point table of the action of $\mathrm{Aut}(D_6) = \langle \varphi_r, \varphi_f, \omega \rangle$ on the conjugacy classes of D_6, where $\omega \in \mathrm{Aut}(D_6)$ is the outer automorphism $f \mapsto rf$ of order 6, that cyclically rotates the axes of reflection.

$\text{cl}(1)$ $\text{cl}(r^3)$ $\text{cl}(r)$ $\text{cl}(r^2)$ $\text{cl}(f)$ $\text{cl}(rf)$

	$\text{cl}(1)$	$\text{cl}(r^3)$	$\text{cl}(r)$	$\text{cl}(r^2)$	$\text{cl}(f)$	$\text{cl}(rf)$
Id						
φ_r						
φ_{r^2}						
φ_f						
φ_{rf}						
φ_{r^2f}						
ω						
$\varphi_r\omega$						
$\varphi_{r^2}\omega$						
$\varphi_f\omega$						
$\varphi_{rf}\omega$						
$\varphi_{r^2f}\omega$						