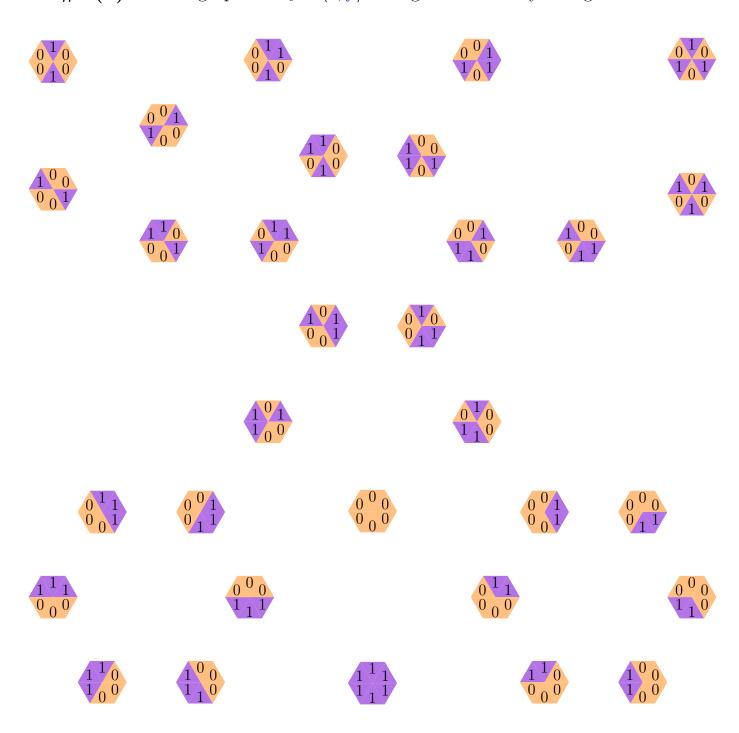
Scratch paper for Abstract Algebra (Math 8510), HW 4

#1(a): Action graph of $D_6 = \langle r, f \rangle$ acting on 31 "binary hexagons."



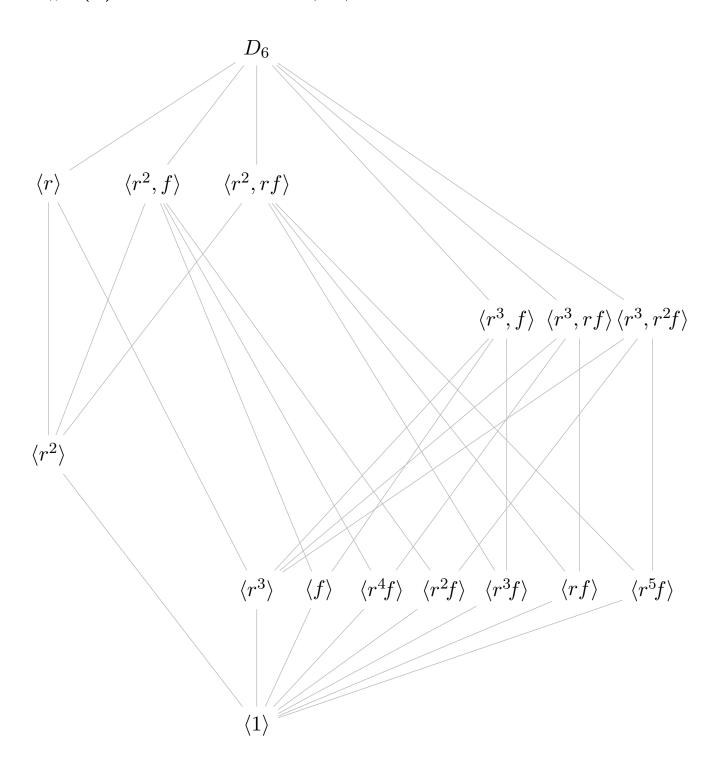
#1(a): Fixed point table of D_6 acting on "binary hexagons."

	$\begin{array}{c} 0 & 0 \\ 0 & 0 \\ \end{array}$	$\frac{1}{1}\frac{1}{1}$	$\begin{smallmatrix}0&1&0\\1&0&1\end{smallmatrix}$	$\begin{array}{c} 1 & 0 & 1 \\ 0 & 1 & 0 \end{array}$	$\begin{smallmatrix}0&0&0\\1&&1\end{smallmatrix}$	$\begin{smallmatrix}0&0&1\\0&1&1\end{smallmatrix}$	$\begin{smallmatrix}0&1&1\\0&&1\\\end{smallmatrix}$	$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$	$\begin{array}{c} 1 & 1 & 0 \\ 1 & 0 & 0 \end{array}$	$\begin{array}{c} 1 & 0 & 0 \\ 1 & 1 & 0 \end{array}$	$\begin{smallmatrix}0&0&0\\1&1&0\end{smallmatrix}$	$\begin{smallmatrix}0&0&0\\0&1&1\end{smallmatrix}$	$\begin{smallmatrix}0&0&1\\0&0&1\end{smallmatrix}$	$\begin{smallmatrix}0&1&1\\0&0&0\end{smallmatrix}$	$\begin{array}{c} 1 1 0 \\ 0 0 0 \end{array}$	100 100
1																
r																
r^2																
r^3																
r^4																
r^5																
f																
rf																
r^2f																
r^3f																
r^4f																
$r^5 f$																

#1(a): Fixed point table of D_6 acting on "binary hexagons" (contin.)

	$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$	$\begin{smallmatrix}1^{0}\\0\\0\end{smallmatrix}$	$001 \\ 100$	$\begin{smallmatrix}1&0&0\\0&1&1\end{smallmatrix}$	$\begin{smallmatrix}0&0&1\\1&0&1\end{smallmatrix}$	$\begin{smallmatrix}0&1&1\\0&1&0\end{smallmatrix}$	$\begin{smallmatrix}1&1&0\\0&0&1\end{smallmatrix}$	$\begin{array}{c} 1 & 0 & 1 \\ 1 & 0 & 0 \end{array}$	$\begin{smallmatrix}0&1&0\\1&1&0\end{smallmatrix}$	$\begin{smallmatrix}0&1&0\\0&1&1\\0&1&1\end{smallmatrix}$	$\begin{smallmatrix}1&0&1\\0&0&1\end{smallmatrix}$	$\begin{smallmatrix}0&1&1\\1&&0\\1&0&0\end{smallmatrix}$	$\begin{array}{c} 1 & 1 & 0 \\ 0 & 1 & 0 \end{array}$	$\begin{array}{c} 1 & 0 \\ 1 & 0 \\ 1 & 0 \end{array}$	$\begin{smallmatrix}0&0&1\\1&1&0\end{smallmatrix}$
1															
r															
r^2															
r^3															
r^4															
r^5															
f															
rf															
$r^2 f$															
r^3f															
r^4f															
$r^5 f$															

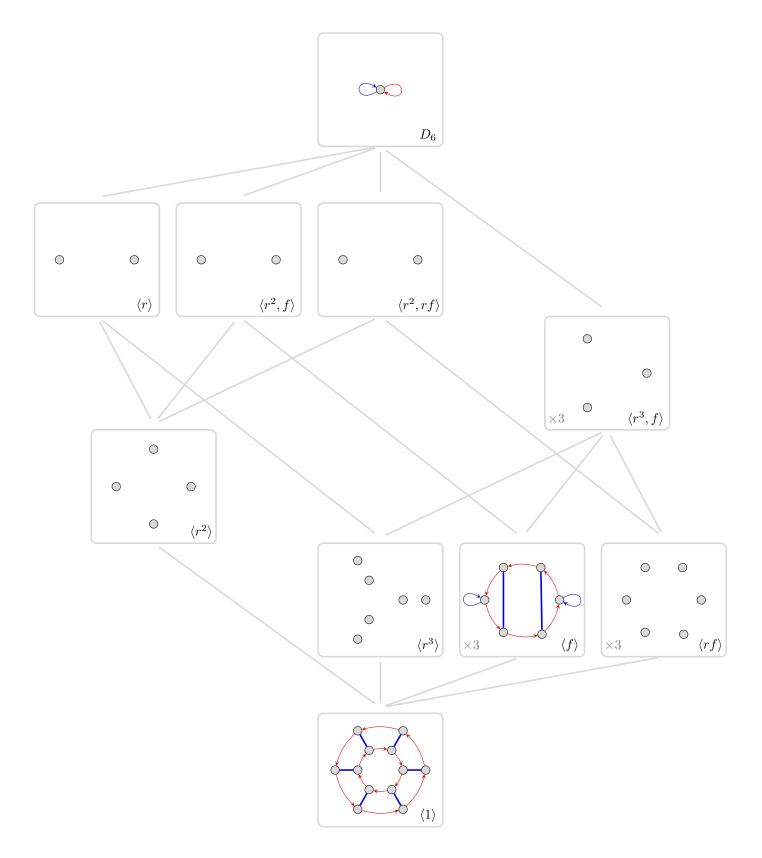
 $\#1(\mathbf{b})$: Action graph of $D_6 = \langle r, f \rangle$ acting on its subgroups by conjugation.



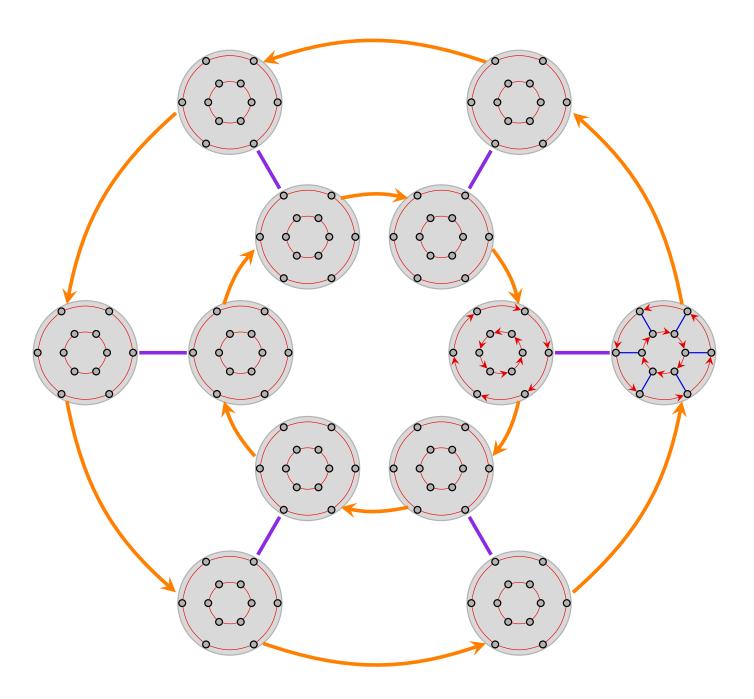
#1(b): Fixed point table of D_6 acting on its subgroups by conjugation.

	D_6	$\langle r \rangle$	$\langle r^2, f \rangle$	$\langle r \rangle$	$r^2, rf \rangle$	$\langle r^3, f$.)	$\langle r^3, rf \rangle$	$\langle r^3, r^2 \rangle$	$^{2}f\rangle$	$\langle r^2 \rangle$	$\langle r^3 \rangle$	$\langle f \rangle$	$\langle r^2 f \rangle$	$\langle r^4 f \rangle$	$\langle rf \rangle$	$\langle r^3 f \rangle$	$\langle r^5 f \rangle$
1																		
r																		
r^2																		
r^3																		
r^4																		
r^5																		
f																		
rf																		
$r^2 f$																		
r^3f																		
r^4f																		
$r^5 f$																		

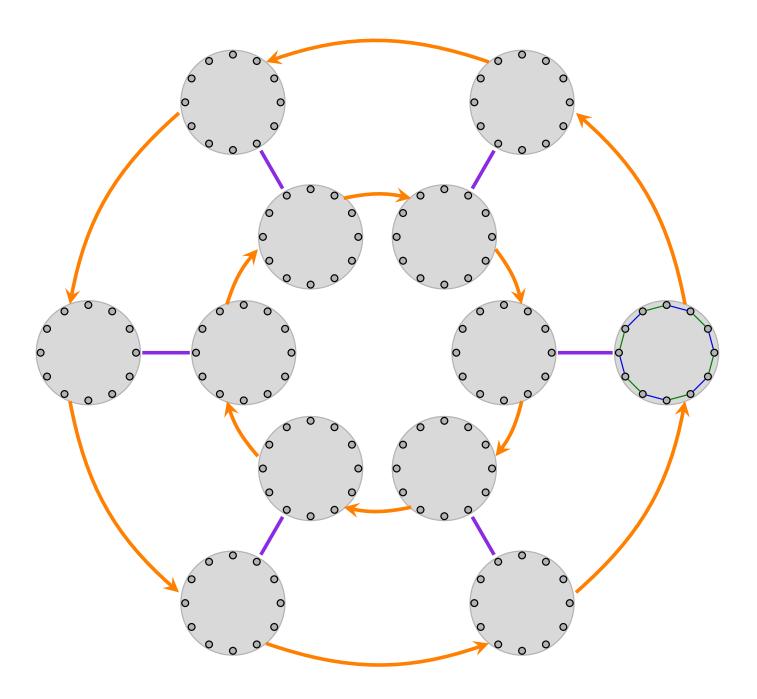
 $\#1(\mathbf{c})$: The action graph poset of $D_6 = \langle r, f \rangle$, constructed by collapsing its Cayley graph by right cosets of its subgroups.



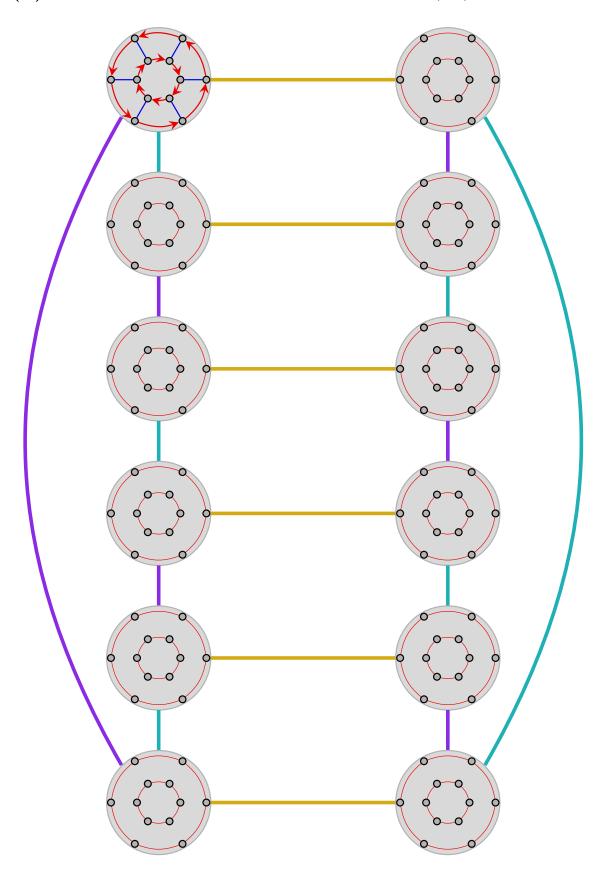
 $\#1(\mathbf{d})$: Cayley graph of the automorphism group $\operatorname{Aut}(D_6) \cong D_6$, with the nodes labeled by rewired copies of the Cayley graph of $D_6 = \langle r, f \rangle$, and with the inner automorphisms labeled.



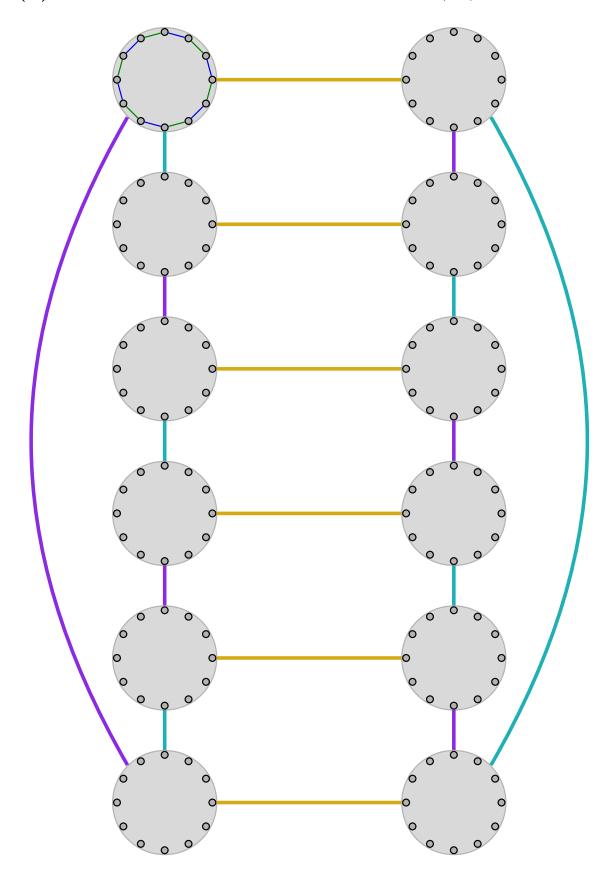
 $\#1(\mathbf{d})$: Cayley graph of the automorphism group $\operatorname{Aut}(D_6) \cong D_6$, with the nodes labeled by rewired copies of the Cayley graph of $D_6 = \langle s, t \rangle = \langle f, rf \rangle$, and with the inner automorphisms labeled.



 $\#1(\mathbf{d})$: Cayley graph of the automorphism group $\mathrm{Aut}(D_6)\cong D_3\rtimes C_2$.



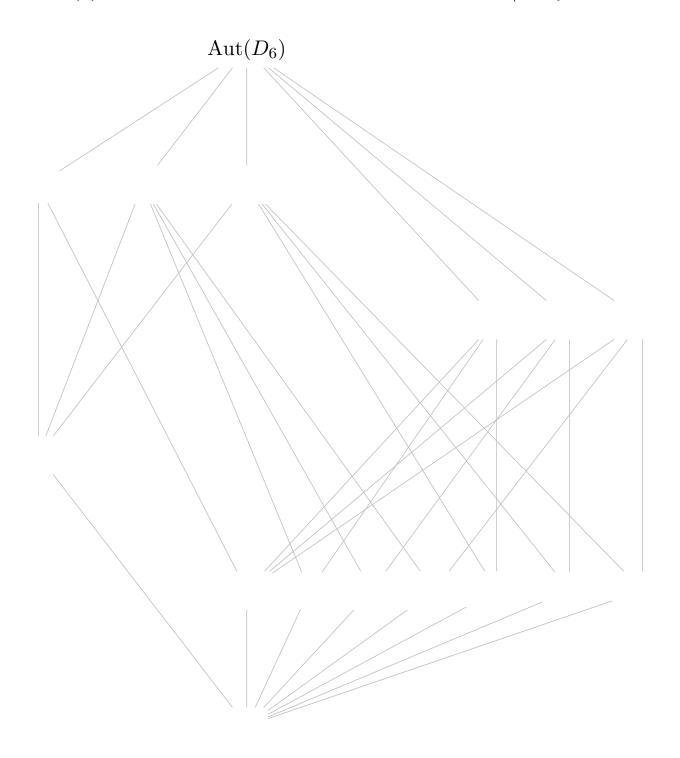
 $\#1(\mathbf{d})$: Cayley graph of the automorphism group $\mathrm{Aut}(D_6)\cong D_3\rtimes C_2$.



#1(e): Partition of $\operatorname{Aut}(D_6) \cong \operatorname{Inn}(D_6) \rtimes \langle \eta \rangle$ into cosets of $\operatorname{Inn}(D_6)$.

		In	$n(D_6)$	$=\langle \varphi$	$_r, \varphi_f \rangle$					Inn($D_6)\omega$			
LJ	Q 1	\bigcap_r	r^2	\int_{f}	\bigcap_{r^2f}	\bigcap_{r^4f}		\bigcap_{1}	r	r^2	f	r^2f	r^4f	
Id	r^3	\bigcap_{r^5}	r^4	\bigcap_{rf}	\bigcap_{r^3f}	\bigcap_{r^5f}		\bigcap_{r^3}	r^5	r^4	rf	r^3f	r^5f	ω
				ı			1							1
$arphi_r$	1	r	r^2	f	r^2f	r^4f			r	r^2	f	r^2f	r^4f	$arphi_r\omega$
41	\bigcap_{r^3}	r^5	r^4	rf	r^3f	$r^5 f$		\bigcap_{r^3}	r^5	r^4	rf	r^3f	$r^5 f$	7700
							_							,
$arphi_{r^2}$	\bigcap_{1}	r	r^2	f	r^2f	r^4f			r	r^2	f	r^2f	r^4f	$arphi_{r^2}\omega$
Υr^2	\bigcap_{r^3}	r^5	r^4	rf	r^3f	$r^5 f$		\bigcap_{r^3}	r^5	r^4	rf	r^3f	$r^5 f$	φ_{r^2}
							_							1
$arphi_f$		r	r^2	f	r^2f	r^4f			r	r^2	f	r^2f	r^4f	$arphi_f \omega$
ΥJ	\bigcap_{r^3}	r^5	r^4	rf	r^3f	$r^5 f$		\bigcap_{r^3}	r^5	r^4	rf	r^3f	$r^5 f$	γ , ω
							7							1
$arphi_{rf}$		r	r^2	f	r^2f	r^4f			r	r^2	f	r^2f	r^4f	$\varphi_{rf}\omega$
PTJ	\bigcap_{r^3}	r^5	r^4	rf	r^3f	$r^5 f$		\bigcap_{r^3}	r^5	r^4	rf	r^3f	$r^5 f$	77750
							_							1
$arphi_{r^2f}$	\bigcap_{1}	r	r^2	f	r^2f	r^4f			r	r^2	f	r^2f	r^4f	$arphi_{r^2f}\omega$
r 17 - 5	()	5.	4	_m f	"3 f	∞5 f		Q	5	4	n f	"3 f	_m 5 f	1 7 7-1

 $\#\mathbf{1}(\mathbf{f})$: Subgroup lattice of $\operatorname{Aut}(D_6) \cong \operatorname{Inn}(D_4) \rtimes \langle \eta \rangle = \langle \varphi_r, \varphi_f, \eta \rangle$.



 $\#1(\mathbf{h})$: Action graph and fixed point table of the action of $\operatorname{Aut}(D_6) = \langle \varphi_r, \varphi_f, \eta \rangle \cong \operatorname{Inn}(D_6) \rtimes \langle \eta \rangle$ on D_6 .

1	r	r^2	f	r^2f	r^4f
r^3	r^5	r^4	rf	$r^3 f$	$r^5\!f$

	1	r	r^2	r^3	r^4	r^5	f	rf	$r^2 f$	$r^3 f$	r^4f	$r^5\!f$
Id												
$arphi_r$												
$arphi_{r^2}$												
$arphi_f$												
$arphi_{rf}$												
$arphi_{r^2\!f}$												
η												
$arphi_r\eta$												
$arphi_{r^2}\eta$												
$arphi_f\eta$												
$arphi_{rf}\eta$												
$arphi_{r^2\!f}\eta$												

 $\#\mathbf{1}(\mathbf{h})$: Action graph and fixed point table of the action of $\operatorname{Aut}(D_6) = \langle \varphi_r, \varphi_f, \omega \rangle$ on D_6 , where $\omega \in \operatorname{Aut}(D_6)$ is the outer automorphism $f \mapsto rf$ of order 6, that cyclically rotates the axes of reflection.

1	r	r^2	f	r^2f	r^4f
r^3	r^5	r^4	rf	r^3f	$r^5\!f$

	1	r	r^2	r^3	r^4	r^5	f	rf	$r^2 f$	r^3f	$r^4 f$	$r^5 f$
Id												
$arphi_r$												
$arphi_{r^2}$												
$arphi_f$												
$arphi_{rf}$												
$arphi_{r^2\!f}$												
ω												
$arphi_r\omega$												
$arphi_{r^2}\omega$												
$arphi_f \omega$												
$arphi_{rf}\omega$												
$arphi_{r^2\!f}\omega$												

 $\#\mathbf{1}(\mathbf{i})$: Action graph and fixed point table of the action of $\operatorname{Aut}(D_6) = \langle \varphi_r, \varphi_f, \eta \rangle \cong \operatorname{Inn}(D_6) \rtimes \langle \eta \rangle$ on the conjugacy classes of D_6 .

(cl(1))

 $(cl(r^3))$

(cl(r))

 $(cl(r^2))$

 $(\operatorname{cl}(f))$

 $\widehat{\operatorname{cl}(rf)}$

	$\operatorname{cl}(1)$	$\operatorname{cl}(r^3)$	$\operatorname{cl}(r)$	$\operatorname{cl}(r^2)$	$\operatorname{cl}(f)$	$\operatorname{cl}(rf)$
Id						
$arphi_r$						
$arphi_{r^2}$						
$arphi_f$						
$arphi_{rf}$						
$arphi_{r^2\!f}$						
η						
$arphi_r\eta$						
$arphi_{r^2}\eta$						
$arphi_f \eta$						
$arphi_{rf}\eta$						
$arphi_{r^2\!f}\eta$						

#1(i): Action graph and fixed point table of the action of $\operatorname{Aut}(D_6) = \langle \varphi_r, \varphi_f, \omega \rangle$ on the conjugacy classes of D_6 , where $\omega \in \operatorname{Aut}(D_6)$ is the outer automorphism $f \mapsto rf$ of order 6, that cyclically rotates the axes of reflection.

	cl(1)	$\operatorname{cl}(r^3)$	$\operatorname{cl}(r)$	$\operatorname{cl}(r^2)$	$\operatorname{cl}(f)$	$\operatorname{cl}(rf)$
Id						
$arphi_r$						
$arphi_{r^2}$						
$arphi_f$						
$arphi_{rf}$						
$arphi_{r^2\!f}$						
ω						
$arphi_r \omega$						
$arphi_{r^2}\omega$						
$arphi_f \omega$						
$arphi_{rf}\omega$						
$arphi_{m}2_{\mathbf{f}}\omega$						