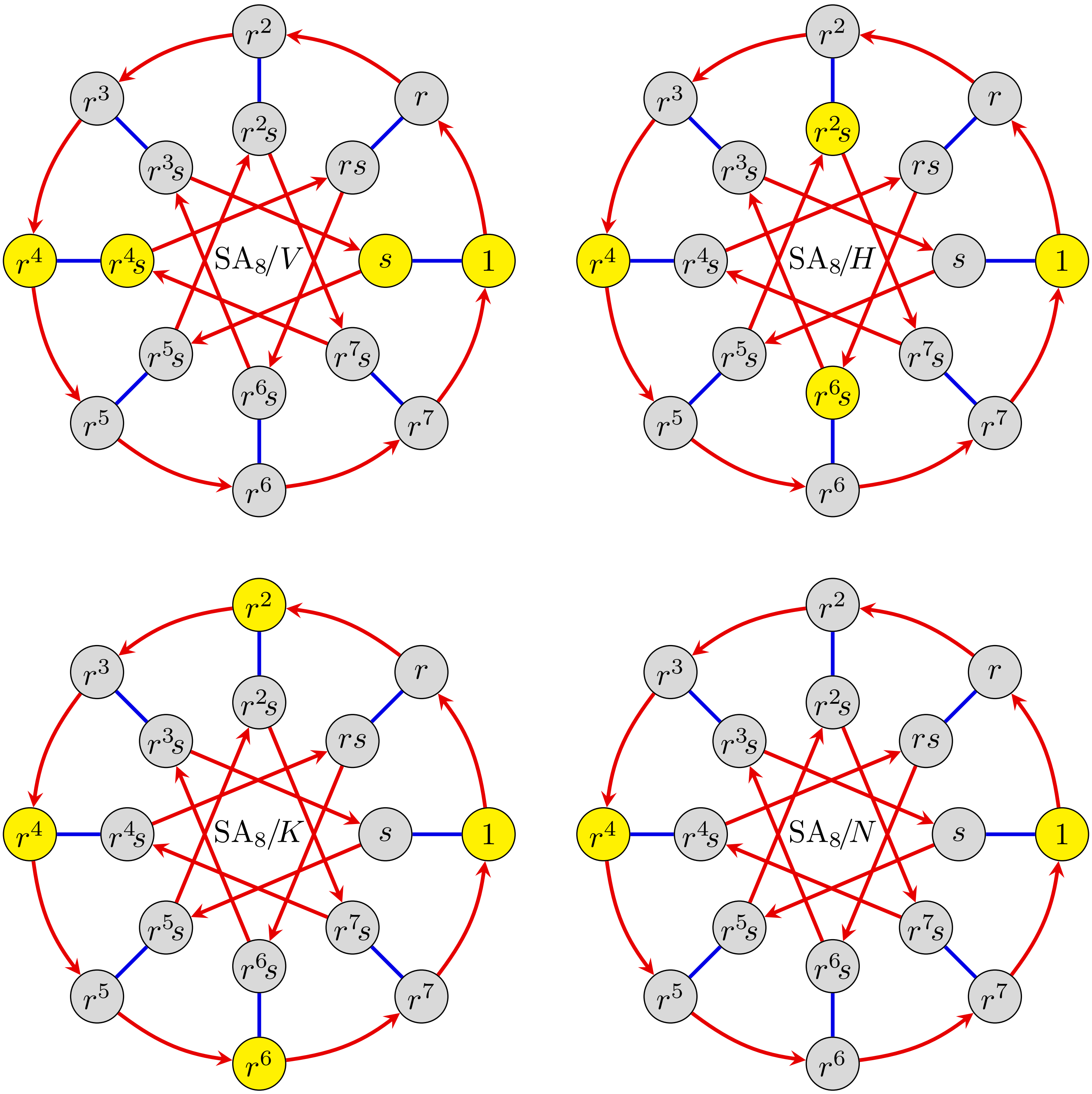
Supplemental material for Math 8510, HW 3

#2(a): Partitions of the *semiabeilan group* SA_8 by cosets of the subgroups $V = \langle r^4, s\rangle$, $H = \langle r^2 s\rangle$, $K = \langle r^2\rangle$, and $N = \langle r^4\rangle$.



#2(b): Cayley tables of the quotients of SA_8 by its subgroups $V = \langle r^4, s\rangle$, $H = \langle r^2 s\rangle$, and $K = \langle r^2\rangle$.

	V			
V				

	H			
H				

	K			
K				

#2(b): The Cayley table of the quotient of SA_8 by its order-2 subgroup $N = \langle r^4 \rangle$.

	N	rN	r^2N	r^3N	sN	rsN	r^2sN	r^3sN
N								
rN								
r^2N								
r^3N								
sN								
rsN								
r^2sN								
r^3sN								

#2(c): "Shoebox diagrams" of the nontrivial proper subgroups of SA_8/N, other than $\langle N\rangle \cong \{e\}$, $\langle rN, sN\rangle \cong \mathrm{SA}_8/N$, and $\langle rN\rangle \cong C_4$ (already done).

r^3N	r^3sN
r^2N	r^2sN
rN	rsN
N	sN

r^3	r^7	r^3s	r^7s
r^2	r^6	r^2s	r^6s
r	r^5	rs	r^5s
1	r^4	s	r^4s

r^3	r^7	r^3s	r^7s
r^2	r^6	r^2s	r^6s
r	r^5	rs	r^5s
1	r^4	s	r^4s

r^3N	r^3sN
r^2N	r^2sN
rN	rsN
N	sN

r^3	r^7	r^3s	r^7s
r^2	r^6	r^2s	r^6s
r	r^5	rs	r^5s
1	r^4	s	r^4s

r^3	r^7	r^3s	r^7s
r^2	r^6	r^2s	r^6s
r	r^5	rs	r^5s
1	r^4	s	r^4s

r^3N	r^3sN
r^2N	r^2sN
rN	rsN
N	sN

r^3	r^7	r^3s	r^7s
r^2	r^6	r^2s	r^6s
r	r^5	rs	r^5s
1	r^4	s	r^4s

r^3	r^7	r^3s	r^7s
r^2	r^6	r^2s	r^6s
r	r^5	rs	r^5s
1	r^4	s	r^4s

r^3N	r^3sN
r^2N	r^2sN
rN	rsN
N	sN

r^3	r^7	r^3s	r^7s
r^2	r^6	r^2s	r^6s
r	r^5	rs	r^5s
1	r^4	s	r^4s

r^3	r^7	r^3s	r^7s
r^2	r^6	r^2s	r^6s
r	r^5	rs	r^5s
1	r^4	s	r^4s

r^3N	r^3sN
r^2N	r^2sN
rN	rsN
N	sN

r^3	r^7	r^3s	r^7s
r^2	r^6	r^2s	r^6s
r	r^5	rs	r^5s
1	r^4	s	r^4s

r^3	r^7	r^3s	r^7s
r^2	r^6	r^2s	r^6s
r	r^5	rs	r^5s
1	r^4	s	r^4s

#2(d): Shoebox diagrams of subgroups of SA_8/V, where $V = \langle r^4, s\rangle$.

r^3V
r^2V
rV
V

r^3	r^7	r^3s	r^7s
r^2	r^6	r^2s	r^6s
r	r^5	rs	r^5s
1	r^4	s	r^4s

r^3	r^7	r^3s	r^7s
r^2	r^6	r^2s	r^6s
r	r^5	rs	r^5s
1	r^4	s	r^4s

r^3V
r^2V
rV
V

r^3	r^7	r^3s	r^7s
r^2	r^6	r^2s	r^6s
r	r^5	rs	r^5s
1	r^4	s	r^4s

r^3	r^7	r^3s	r^7s
r^2	r^6	r^2s	r^6s
r	r^5	rs	r^5s
1	r^4	s	r^4s

r^3V
r^2V
rV
V

r^3	r^7	r^3s	r^7s
r^2	r^6	r^2s	r^6s
r	r^5	rs	r^5s
1	r^4	s	r^4s

r^3	r^7	r^3s	r^7s
r^2	r^6	r^2s	r^6s
r	r^5	rs	r^5s
1	r^4	s	r^4s

#1(d): Shoebox diagrams of subgroups of SA_8/H, where $H = \langle r^2 s \rangle$.

r^3H
r^2H
rH
H

r^3	r^5s	r^7	rs
r^2	r^4s	r^6	s
r	r^3s	r^5	r^7s
1	r^2s	r^4	r^6s

r^3	r^5s	r^7	rs
r^2	r^4s	r^6	s
r	r^3s	r^5	r^7s
1	r^2s	r^4	r^6s

r^3H
r^2H
rH
H

r^3	r^5s	r^7	rs
r^2	r^4s	r^6	s
r	r^3s	r^5	r^7s
1	r^2s	r^4	r^6s

r^3	r^5s	r^7	rs
r^2	r^4s	r^6	s
r	r^3s	r^5	r^7s
1	r^2s	r^4	r^6s

r^3H
r^2H
rH
H

r^3	r^5s	r^7	rs
r^2	r^4s	r^6	s
r	r^3s	r^5	r^7s
1	r^2s	r^4	r^6s

r^3	r^5s	r^7	rs
r^2	r^4s	r^6	s
r	r^3s	r^5	r^7s
1	r^2s	r^4	r^6s

#1(d): Shoebox diagrams of subgroups of SA_8/K, where $K = \langle r^2 \rangle$.

rsK
sK
rK
K

rs	r^3s	r^5s	r^7s
s	r^2s	r^4s	r^6s
r	r^3	r^5	r^7
1	r^2	r^4	r^6

rs	r^3s	r^5s	r^7s
s	r^2s	r^4s	r^6s
r	r^3	r^5	r^7
1	r^2	r^4	r^6

rsK
sK
rK
K

rs	r^3s	r^5s	r^7s
s	r^2s	r^4s	r^6s
r	r^3	r^5	r^7
1	r^2	r^4	r^6

rs	r^3s	r^5s	r^7s
s	r^2s	r^4s	r^6s
r	r^3	r^5	r^7
1	r^2	r^4	r^6

rsK
sK
rK
K

rs	r^3s	r^5s	r^7s
s	r^2s	r^4s	r^6s
r	r^3	r^5	r^7
1	r^2	r^4	r^6

rs	r^3s	r^5s	r^7s
s	r^2s	r^4s	r^6s
r	r^3	r^5	r^7
1	r^2	r^4	r^6

rsK
sK
rK
K

rs	r^3s	r^5s	r^7s
s	r^2s	r^4s	r^6s
r	r^3	r^5	r^7
1	r^2	r^4	r^6

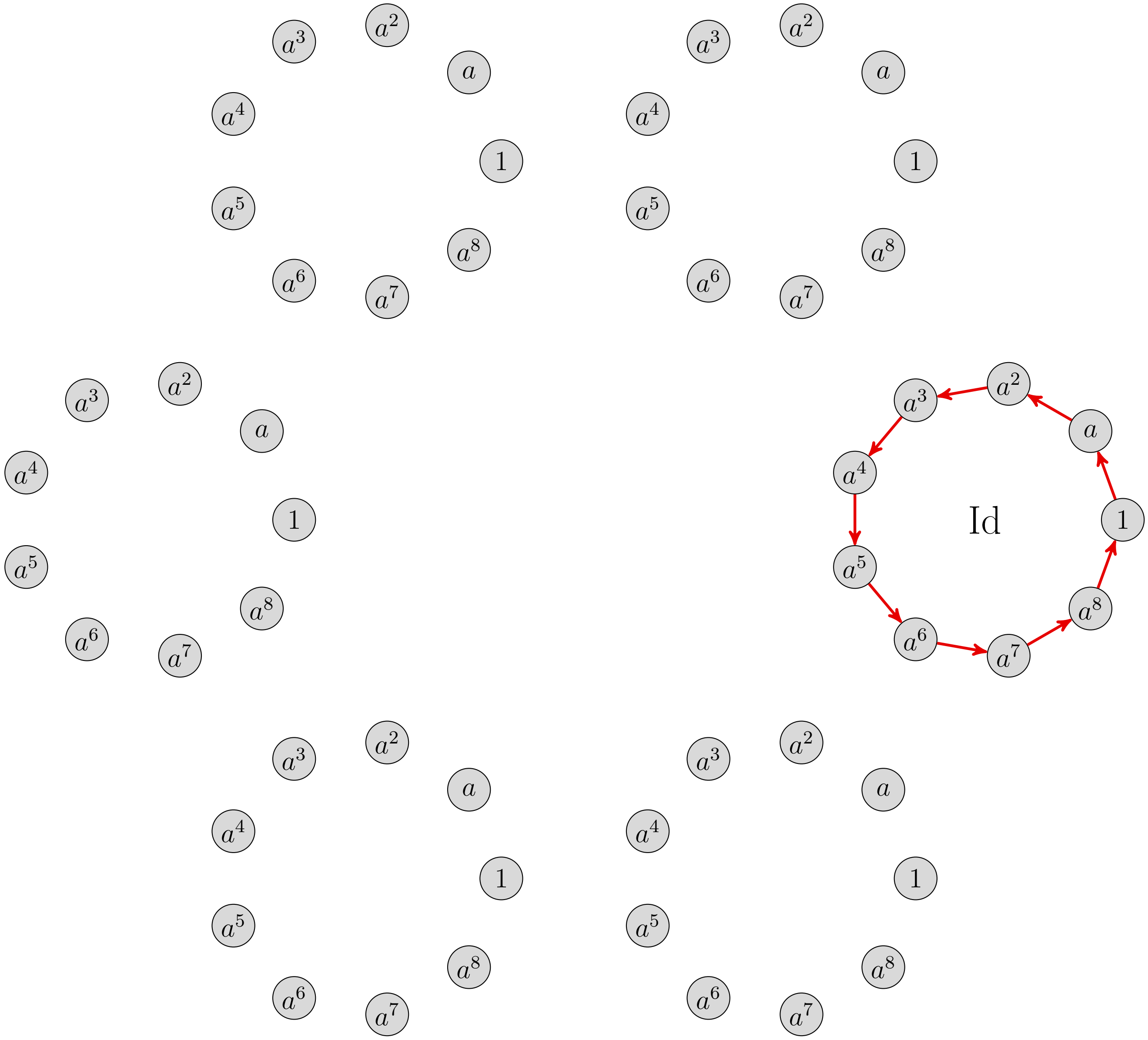
rs	r^3s	r^5s	r^7s
s	r^2s	r^4s	r^6s
r	r^3	r^5	r^7
1	r^2	r^4	r^6

rsK
sK
rK
K

rs	r^3s	r^5s	r^7s
s	r^2s	r^4s	r^6s
r	r^3	r^5	r^7
1	r^2	r^4	r^6

rs	r^3s	r^5s	r^7s
s	r^2s	r^4s	r^6s
r	r^3	r^5	r^7
1	r^2	r^4	r^6

#5: Cayley graph of $\mathrm{Aut}(C_9)$, the *automorphism group* of C_9, which is isomorphic to $U_9 = \{1, 2, 4, 5, 7, 8\}$, the multiplitive group of integers modulo 9. The nodes are labeled by rewirings (automorphisms) of the Cayley diagram.



#5: Cayley graph of $C_9 \rtimes C_3 = \langle a, b\rangle = \left\{a^i b^j \mid 0 \le i \le 8,\ 0 \le j \le 2\right\}$.