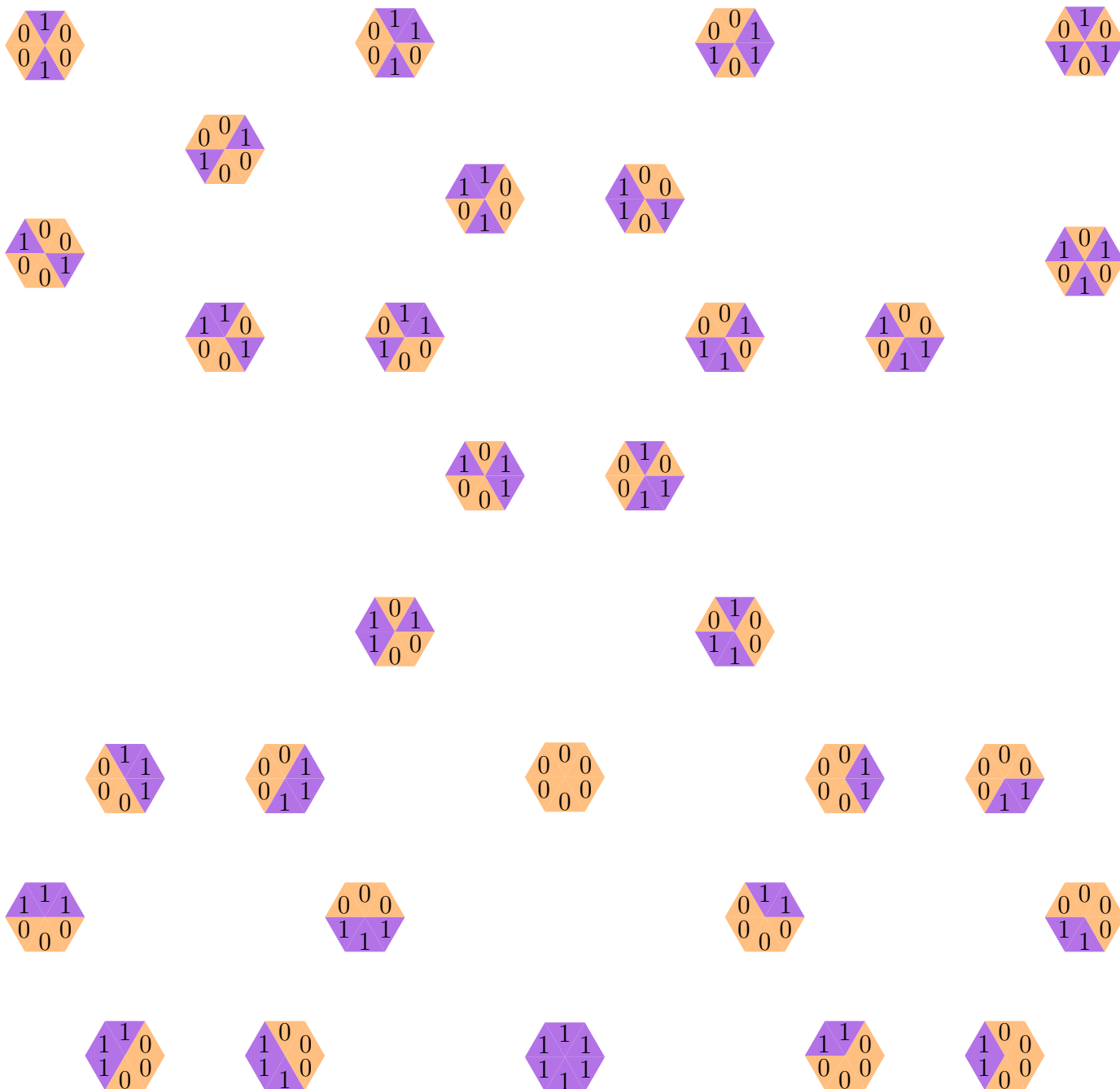


# Supplemental material for Abstract Algebra (Math 8510), HW 4

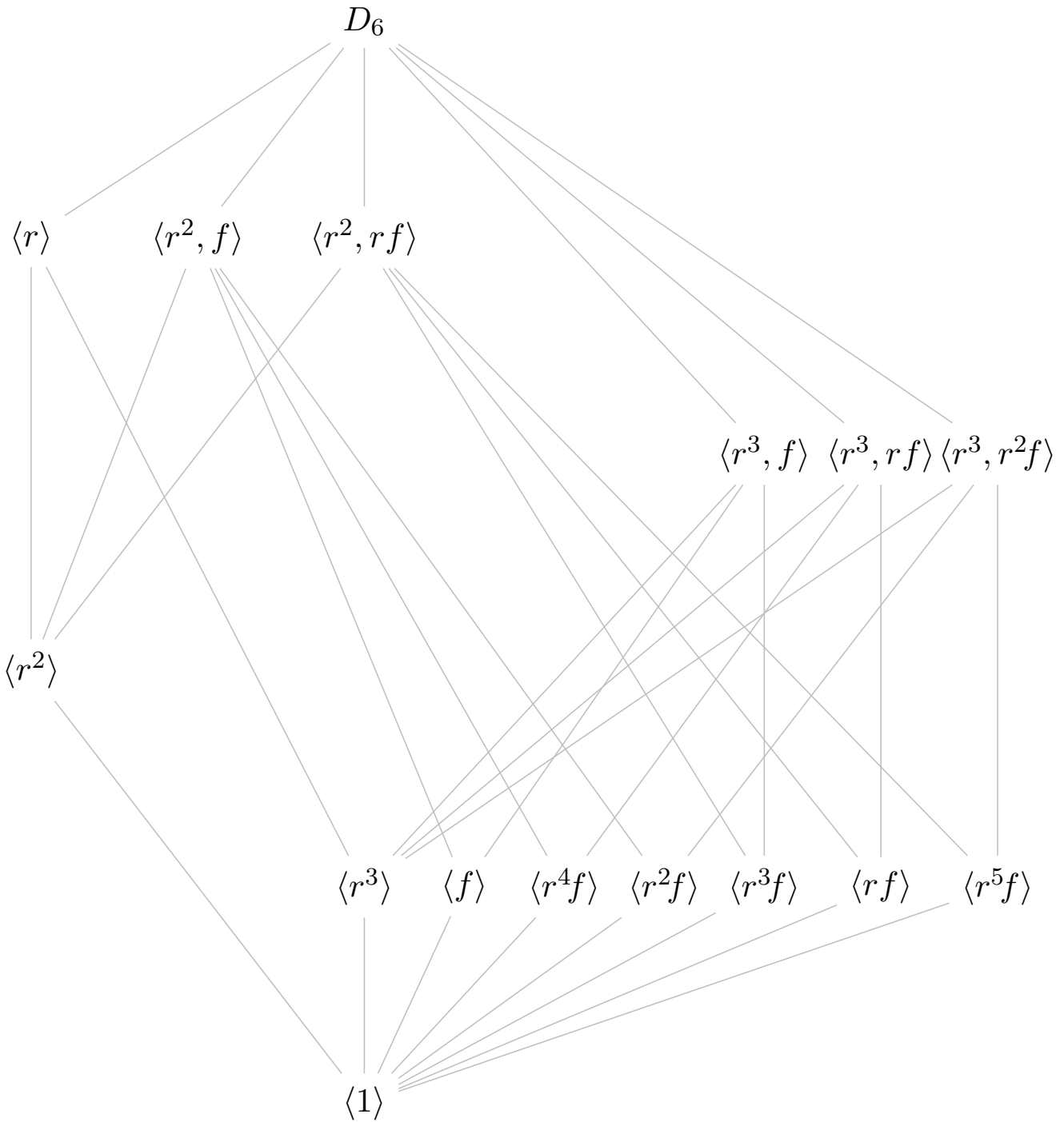
#1(a): Action graph of  $D_6 = \langle r, f \rangle$  acting on 31 “binary hexagons.”





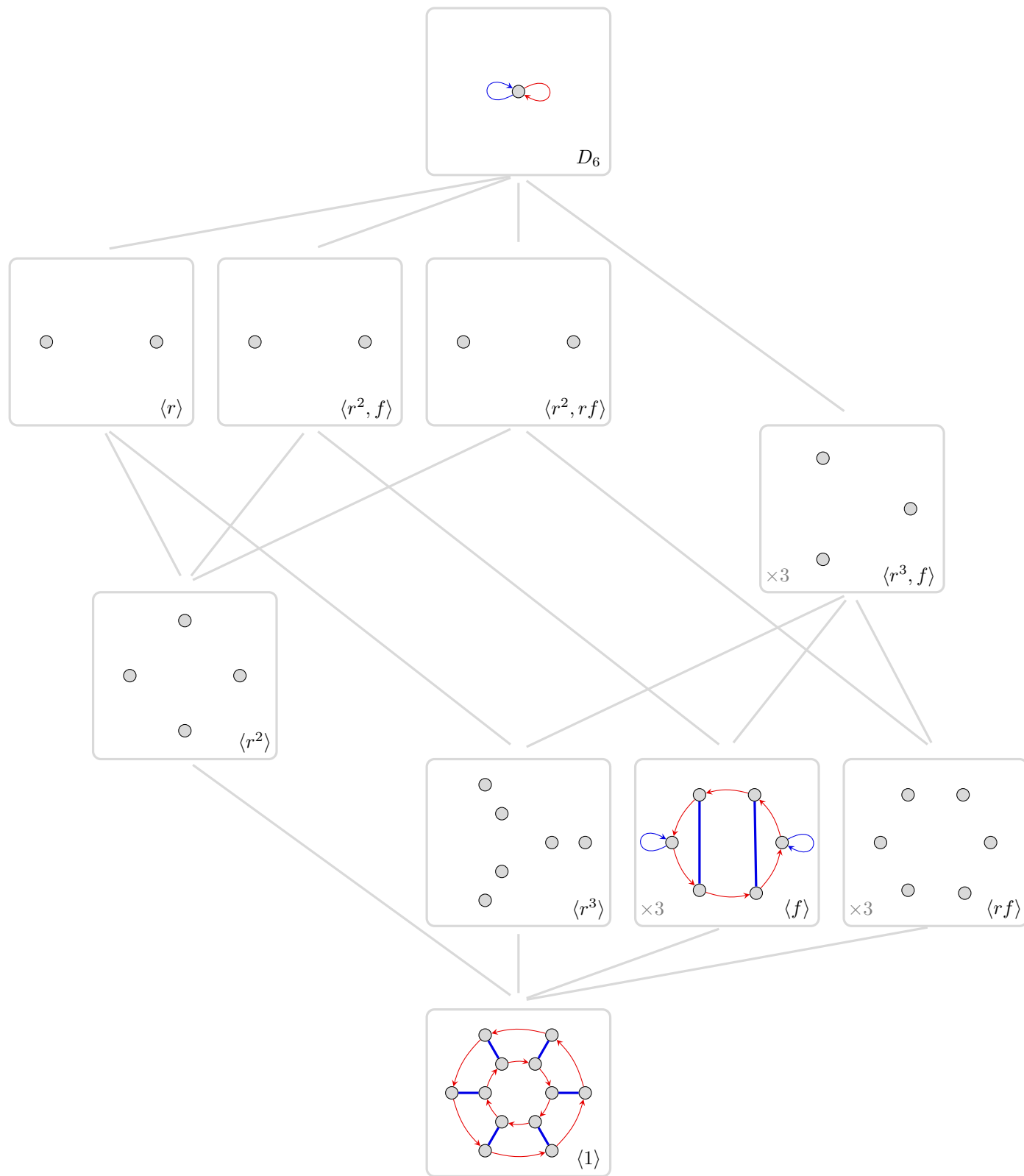


#1(b): Action graph of  $D_6 = \langle r, f \rangle$  acting on its subgroups by conjugation.

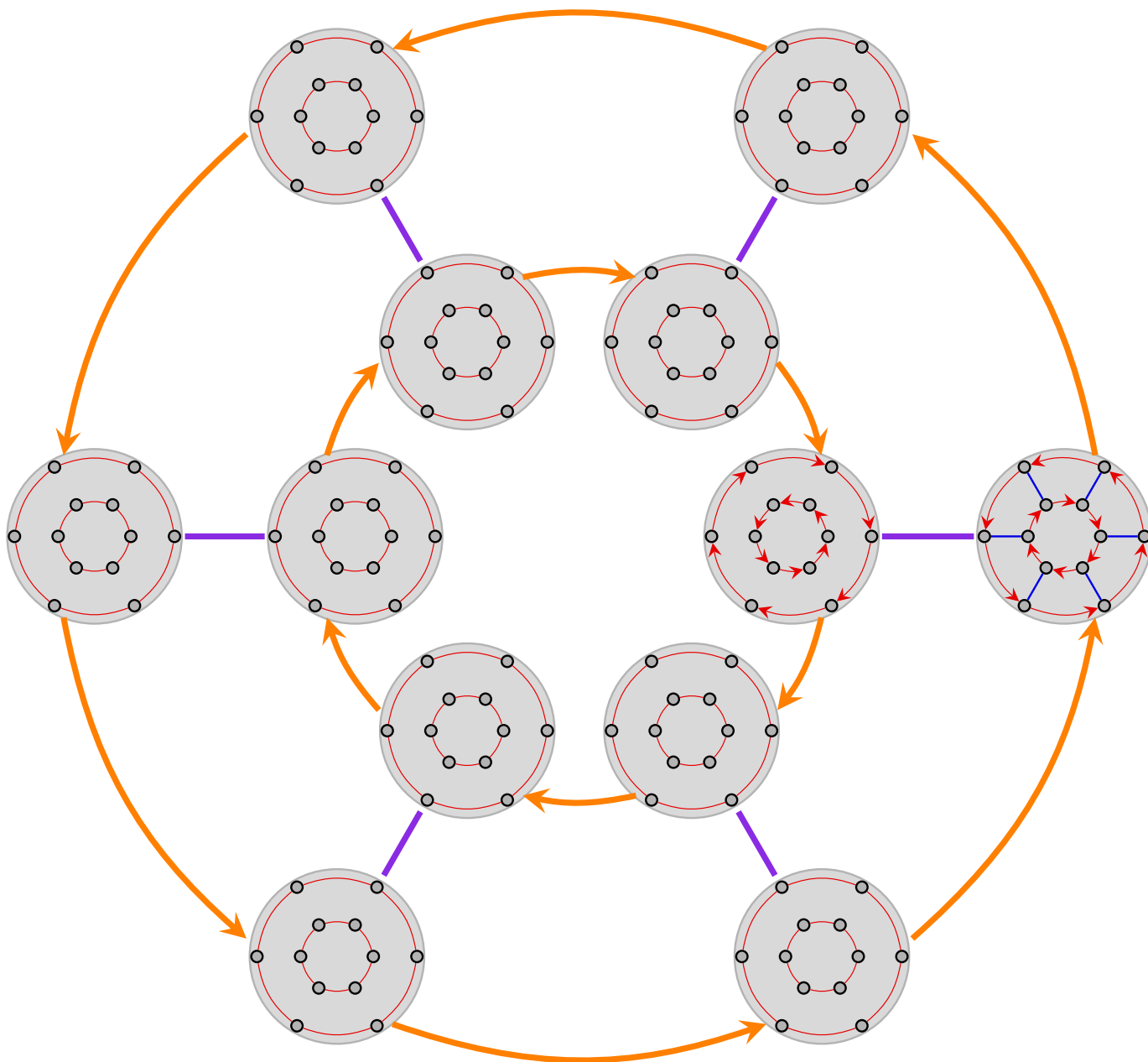




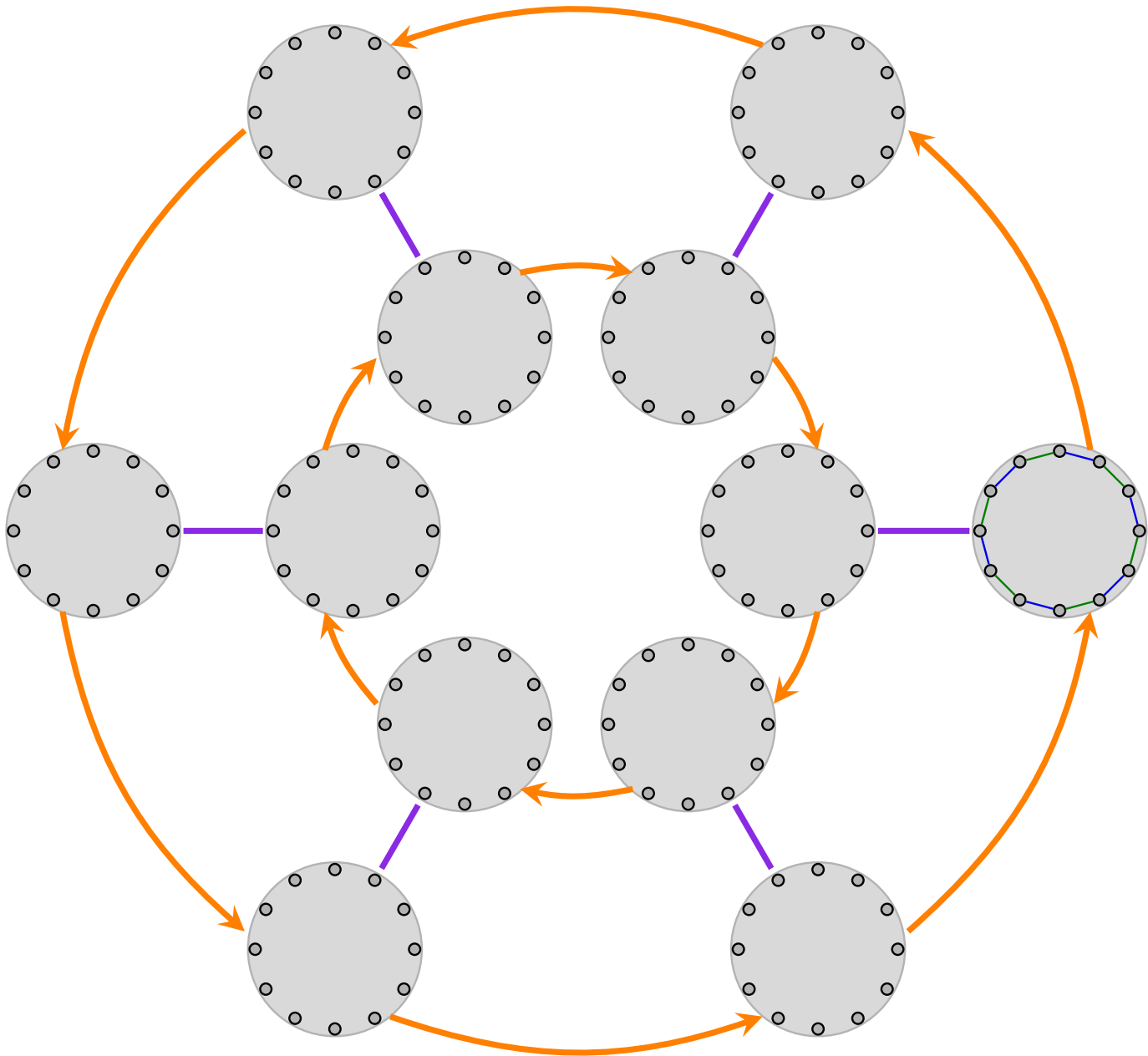
#1(c): The action graph poset of  $D_6 = \langle r, f \rangle$ , constructed by collapsing its Cayley graph by right cosets of its subgroups.



**#1(d)**: Cayley graph of the automorphism group  $\text{Aut}(D_6) \cong D_6$ , with the nodes labeled by rewired copies of the Cayley graph of  $D_6 = \langle r, f \rangle$ , and with the inner automorphisms labeled.

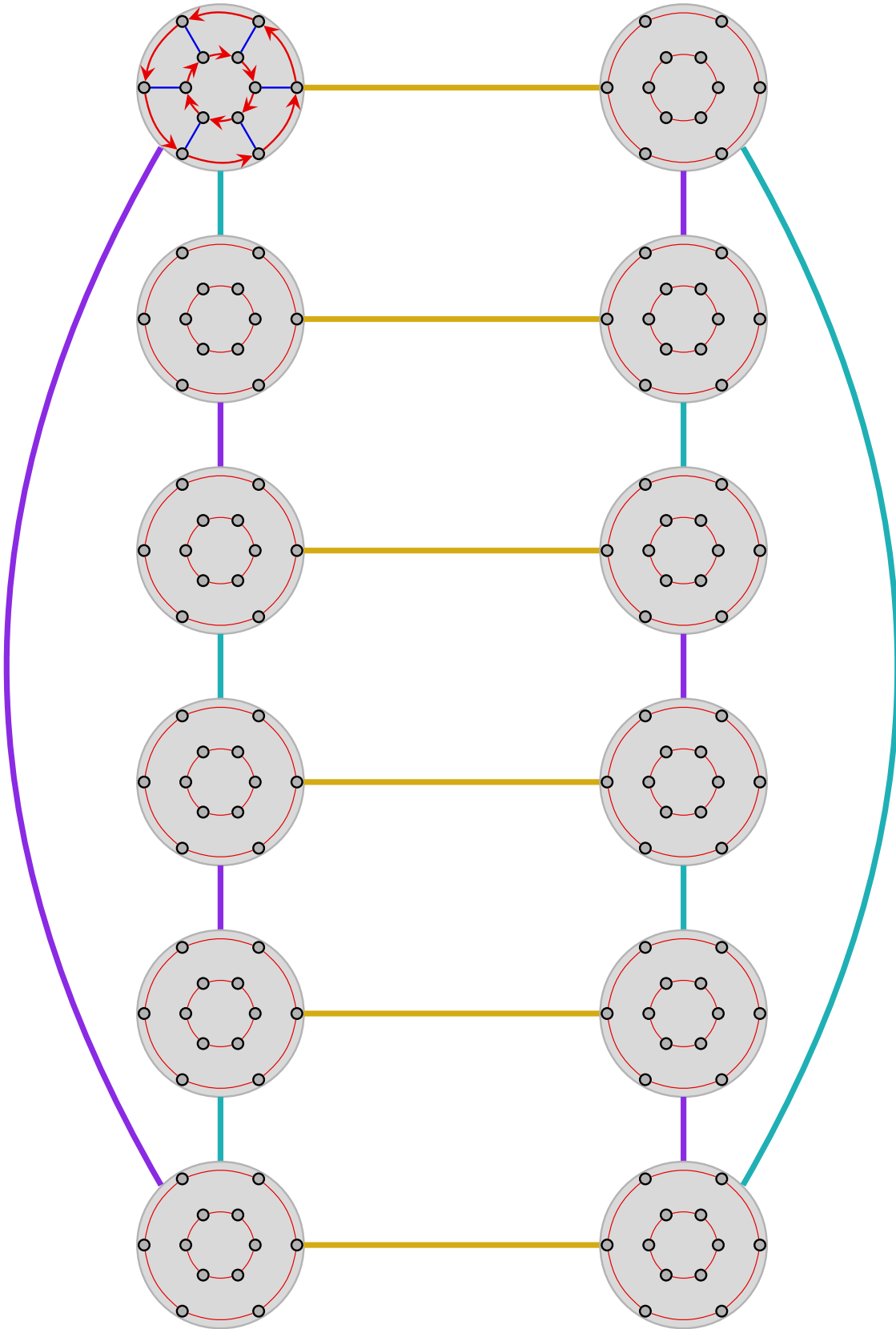


**#1(d)**: Cayley graph of the automorphism group  $\text{Aut}(D_6) \cong D_6$ , with the nodes labeled by rewired copies of the Cayley graph of  $D_6 = \langle s, t \rangle = \langle f, rf \rangle$ , and with the inner automorphisms labeled.

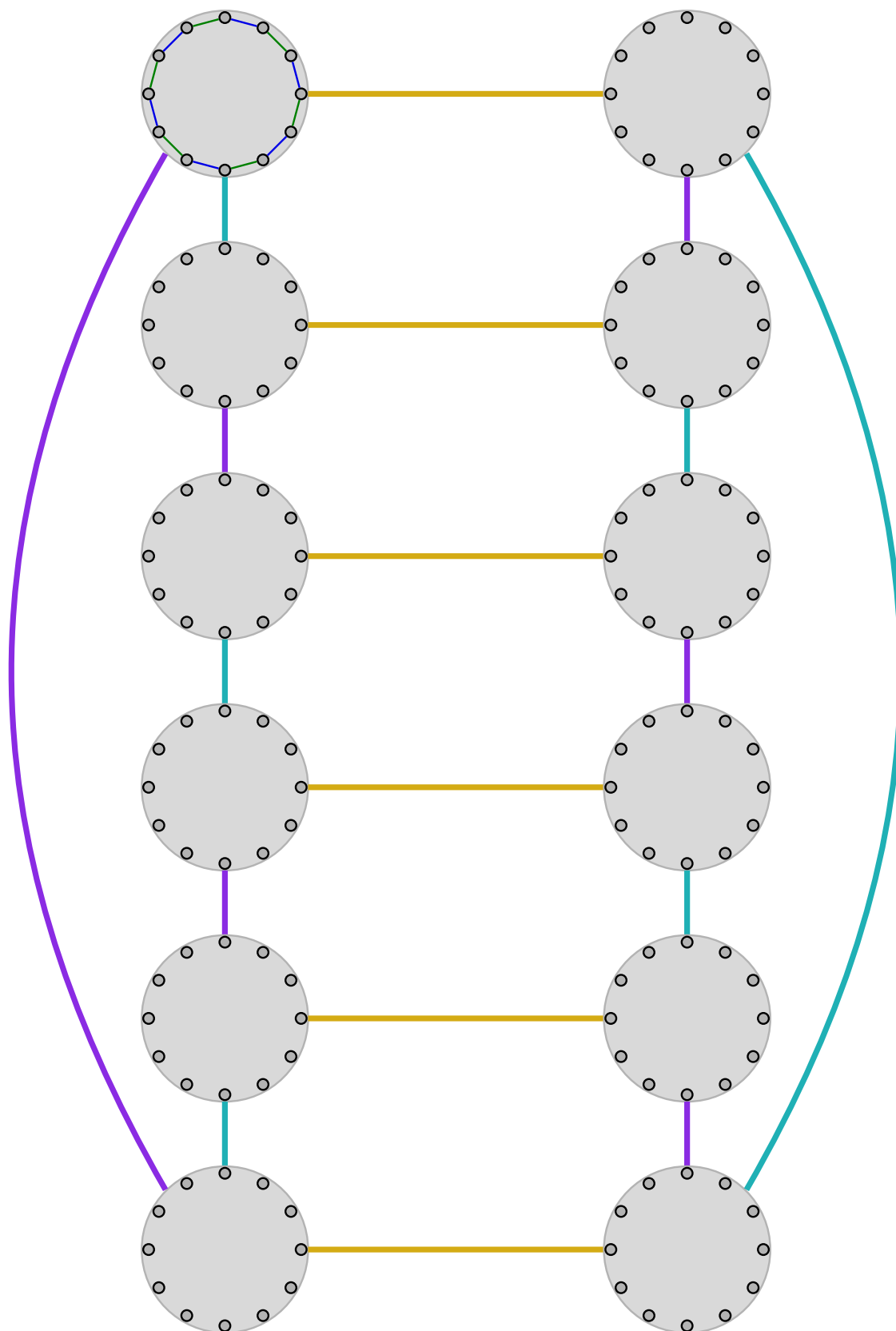




#1(d): Cayley graph of the automorphism group  $\text{Aut}(D_6) \cong D_3 \times C_2$ .



#1(d): Cayley graph of the automorphism group  $\text{Aut}(D_6) \cong D_3 \rtimes C_2$ .



#1(e): Partition of  $\text{Aut}(D_6) \cong \text{Inn}(D_6) \rtimes \langle \eta \rangle$  into cosets of  $\text{Inn}(D_6)$ .

$\text{Inn}(D_6) = \langle \varphi_r, \varphi_f \rangle$

$\text{Inn}(D_6)\omega$

Id

$\begin{array}{c} \curvearrowright \\ 1 \end{array}$	$\begin{array}{c} \curvearrowright \\ r \end{array}$	$\begin{array}{c} \curvearrowright \\ r^2 \end{array}$	$\begin{array}{c} \curvearrowright \\ f \end{array}$	$\begin{array}{c} \curvearrowright \\ r^2 f \end{array}$	$\begin{array}{c} \curvearrowright \\ r^4 f \end{array}$
$\begin{array}{c} \curvearrowright \\ r^3 \end{array}$	$\begin{array}{c} \curvearrowright \\ r^5 \end{array}$	$\begin{array}{c} \curvearrowright \\ r^4 \end{array}$	$\begin{array}{c} \curvearrowright \\ r f \end{array}$	$\begin{array}{c} \curvearrowright \\ r^3 f \end{array}$	$\begin{array}{c} \curvearrowright \\ r^5 f \end{array}$

$\begin{array}{c} \curvearrowright \\ 1 \end{array}$	$r$	$r^2$	$f$	$r^2 f$	$r^4 f$
$\begin{array}{c} \curvearrowright \\ r^3 \end{array}$	$r^5$	$r^4$	$r f$	$r^3 f$	$r^5 f$

$\omega$

$\varphi_r$

$\begin{array}{c} \curvearrowright \\ 1 \end{array}$	$r$	$r^2$	$f$	$r^2 f$	$r^4 f$
$\begin{array}{c} \curvearrowright \\ r^3 \end{array}$	$r^5$	$r^4$	$r f$	$r^3 f$	$r^5 f$

$\begin{array}{c} \curvearrowright \\ 1 \end{array}$	$r$	$r^2$	$f$	$r^2 f$	$r^4 f$
$\begin{array}{c} \curvearrowright \\ r^3 \end{array}$	$r^5$	$r^4$	$r f$	$r^3 f$	$r^5 f$

$\varphi_r \omega$

$\varphi_{r^2}$

$\begin{array}{c} \curvearrowright \\ 1 \end{array}$	$r$	$r^2$	$f$	$r^2 f$	$r^4 f$
$\begin{array}{c} \curvearrowright \\ r^3 \end{array}$	$r^5$	$r^4$	$r f$	$r^3 f$	$r^5 f$

$\begin{array}{c} \curvearrowright \\ 1 \end{array}$	$r$	$r^2$	$f$	$r^2 f$	$r^4 f$
$\begin{array}{c} \curvearrowright \\ r^3 \end{array}$	$r^5$	$r^4$	$r f$	$r^3 f$	$r^5 f$

$\varphi_{r^2} \omega$

$\varphi_f$

$\begin{array}{c} \curvearrowright \\ 1 \end{array}$	$r$	$r^2$	$f$	$r^2 f$	$r^4 f$
$\begin{array}{c} \curvearrowright \\ r^3 \end{array}$	$r^5$	$r^4$	$r f$	$r^3 f$	$r^5 f$

$\begin{array}{c} \curvearrowright \\ 1 \end{array}$	$r$	$r^2$	$f$	$r^2 f$	$r^4 f$
$\begin{array}{c} \curvearrowright \\ r^3 \end{array}$	$r^5$	$r^4$	$r f$	$r^3 f$	$r^5 f$

$\varphi_f \omega$

$\varphi_{r f}$

$\begin{array}{c} \curvearrowright \\ 1 \end{array}$	$r$	$r^2$	$f$	$r^2 f$	$r^4 f$
$\begin{array}{c} \curvearrowright \\ r^3 \end{array}$	$r^5$	$r^4$	$r f$	$r^3 f$	$r^5 f$

$\begin{array}{c} \curvearrowright \\ 1 \end{array}$	$r$	$r^2$	$f$	$r^2 f$	$r^4 f$
$\begin{array}{c} \curvearrowright \\ r^3 \end{array}$	$r^5$	$r^4$	$r f$	$r^3 f$	$r^5 f$

$\varphi_{r f} \omega$

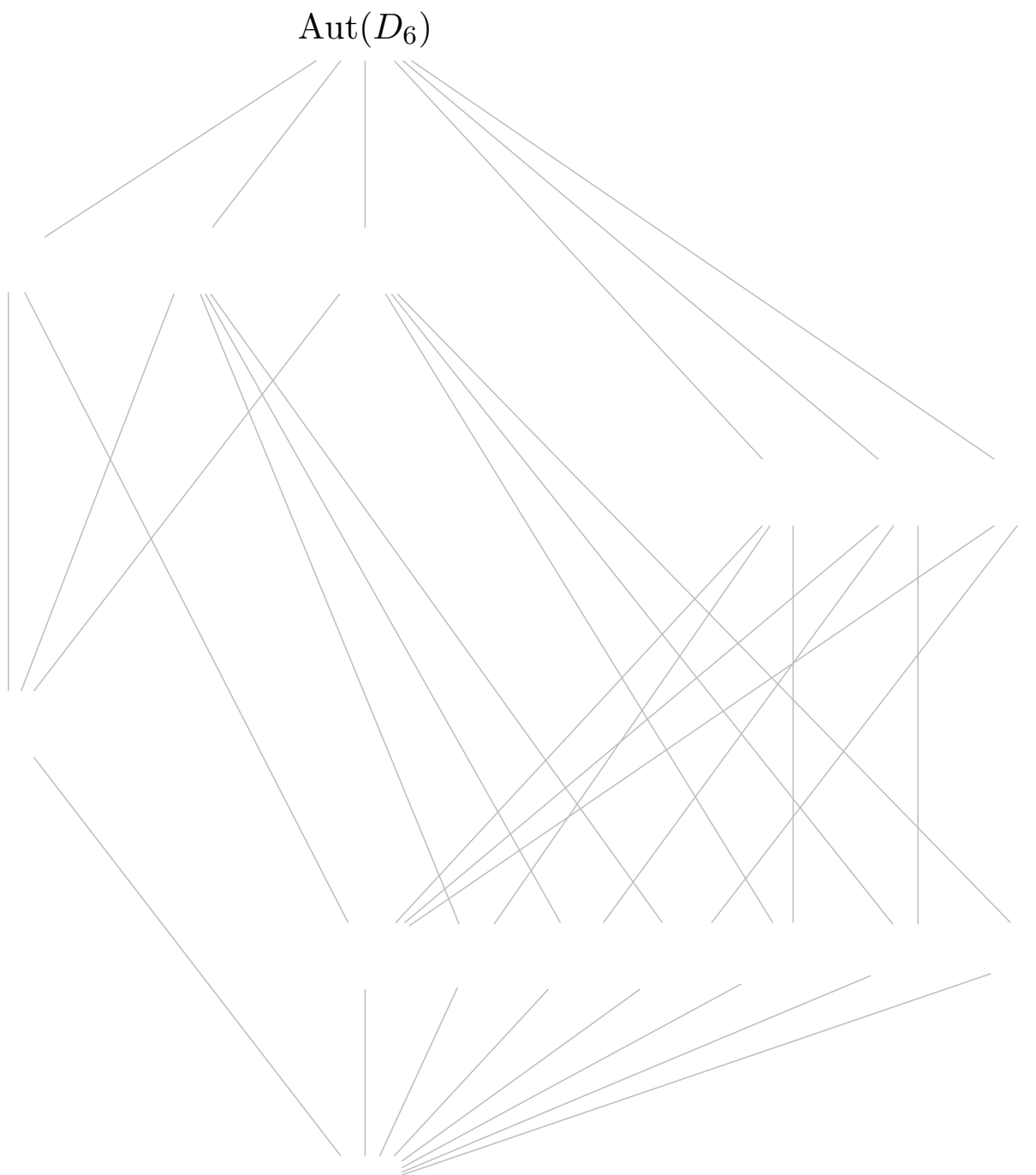
$\varphi_{r^2 f}$

$\begin{array}{c} \curvearrowright \\ 1 \end{array}$	$r$	$r^2$	$f$	$r^2 f$	$r^4 f$
$\begin{array}{c} \curvearrowright \\ r^3 \end{array}$	$r^5$	$r^4$	$r f$	$r^3 f$	$r^5 f$

$\begin{array}{c} \curvearrowright \\ 1 \end{array}$	$r$	$r^2$	$f$	$r^2 f$	$r^4 f$
$\begin{array}{c} \curvearrowright \\ r^3 \end{array}$	$r^5$	$r^4$	$r f$	$r^3 f$	$r^5 f$

$\varphi_{r^2 f} \omega$

#1(f): Subgroup lattice of  $\text{Aut}(D_6) \cong \text{Inn}(D_4) \rtimes \langle \eta \rangle = \langle \varphi_r, \varphi_f, \eta \rangle$ .







**#1(i)**: Action graph and fixed point table of the action of  $\text{Aut}(D_6) = \langle \varphi_r, \varphi_f, \eta \rangle \cong \text{Inn}(D_6) \rtimes \langle \eta \rangle$  on the conjugacy classes of  $D_6$ .



	$\text{cl}(1)$	$\text{cl}(r^3)$	$\text{cl}(r)$	$\text{cl}(r^2)$	$\text{cl}(f)$	$\text{cl}(rf)$
Id						
$\varphi_r$						
$\varphi_{r^2}$						
$\varphi_f$						
$\varphi_{rf}$						
$\varphi_{r^2f}$						
$\eta$						
$\varphi_r\eta$						
$\varphi_{r^2}\eta$						
$\varphi_f\eta$						
$\varphi_{rf}\eta$						
$\varphi_{r^2f}\eta$						

**#1(i)**: Action graph and fixed point table of the action of  $\text{Aut}(D_6) = \langle \varphi_r, \varphi_f, \omega \rangle$  on the conjugacy classes of  $D_6$ , where  $\omega \in \text{Aut}(D_6)$  is the outer automorphism  $f \mapsto rf$  of order 6, that cyclically rotates the axes of reflection.



	cl(1)	cl( $r^3$ )	cl( $r$ )	cl( $r^2$ )	cl( $f$ )	cl( $rf$ )
Id						
$\varphi_r$						
$\varphi_{r^2}$						
$\varphi_f$						
$\varphi_{rf}$						
$\varphi_{r^2f}$						
$\omega$						
$\varphi_r\omega$						
$\varphi_{r^2}\omega$						
$\varphi_f\omega$						
$\varphi_{rf}\omega$						
$\varphi_{r^2f}\omega$						