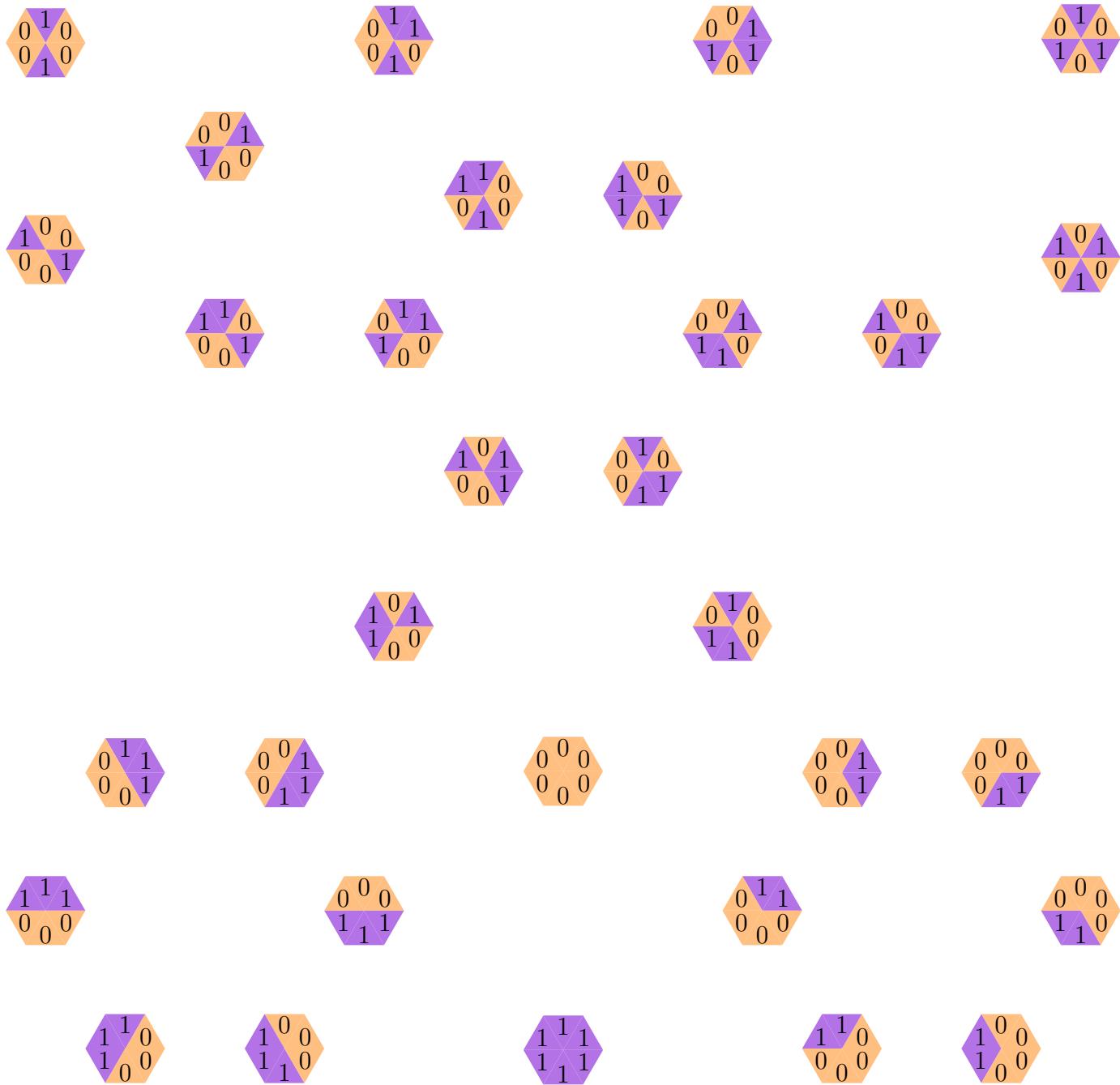


Supplemental material for Abstract Algebra (Math 8510), HW 4

#1(a): Action graph of $D_6 = \langle r, f \rangle$ acting on 31 “binary hexagons.”



#1(a): Fixed point table of D_6 acting on “binary hexagons.”

A diagram showing a sequence of 16 hexagonal tiles arranged in a horizontal row. Each hexagon contains a binary string of length 6. The strings are:

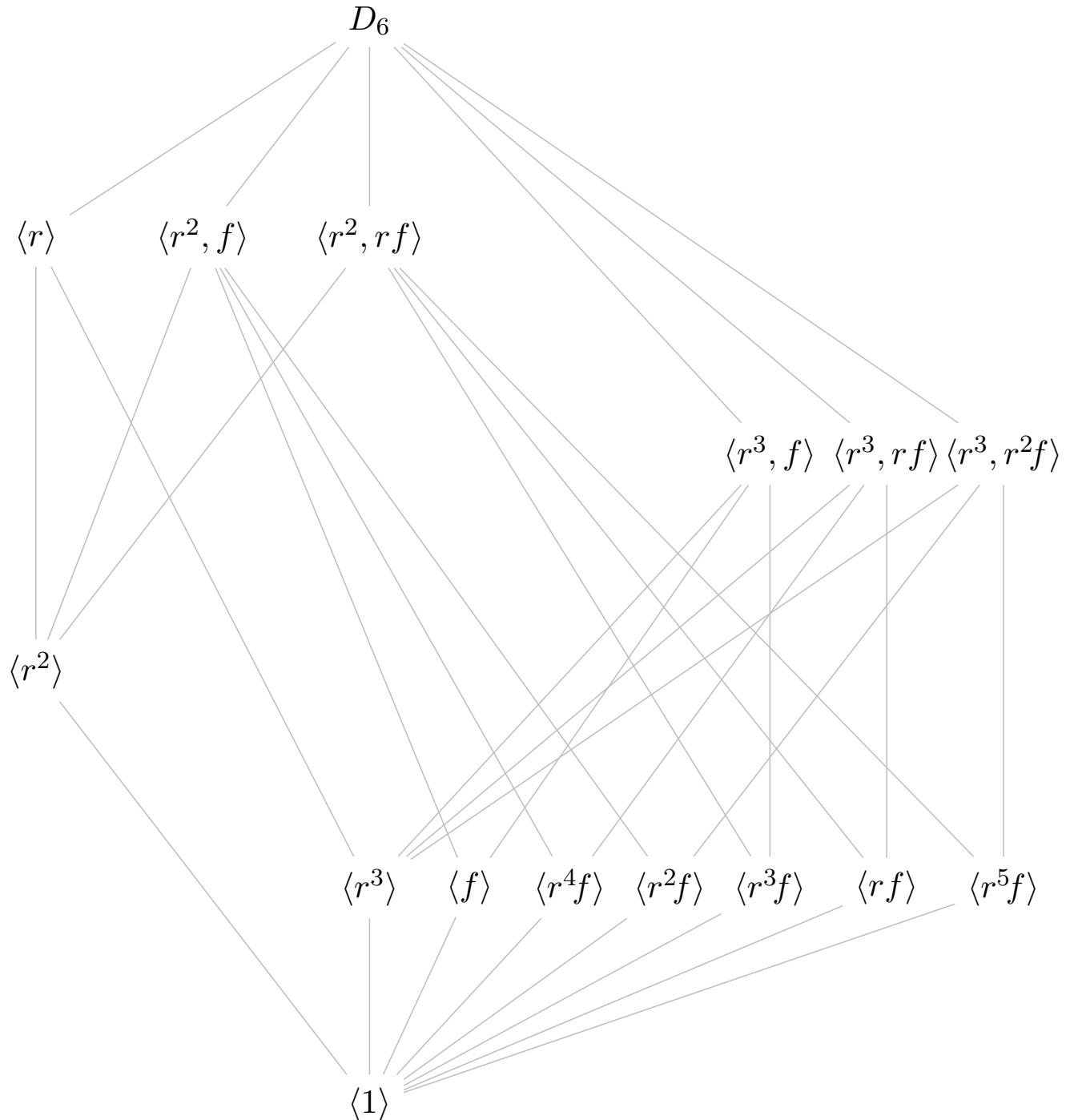
- 000
- 111
- 010
- 101
- 000
- 111
- 001
- 011
- 010
- 110
- 000
- 000
- 001
- 000
- 111
- 000
- 110

The vertical axis on the left lists powers of r :

- 1
- r
- r^2
- r^3
- r^4
- r^5
- f
- rf
- r^2f
- r^3f
- r^4f
- r^5f

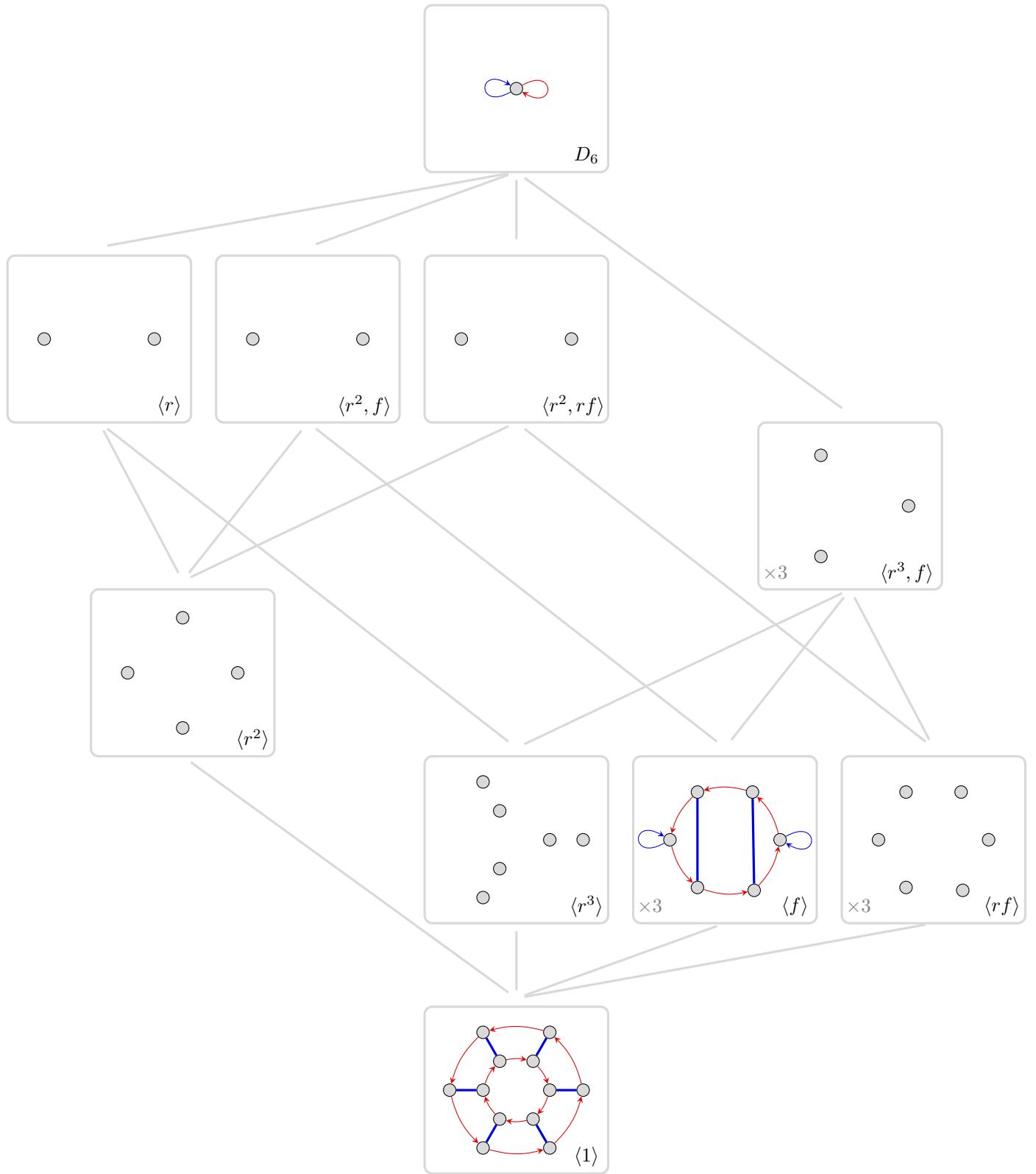
#1(a): Fixed point table of D_6 acting on “binary hexagons” (contin.)

#1(b): Action graph of $D_6 = \langle \textcolor{red}{r}, \textcolor{blue}{f} \rangle$ acting on its subgroups by conjugation.

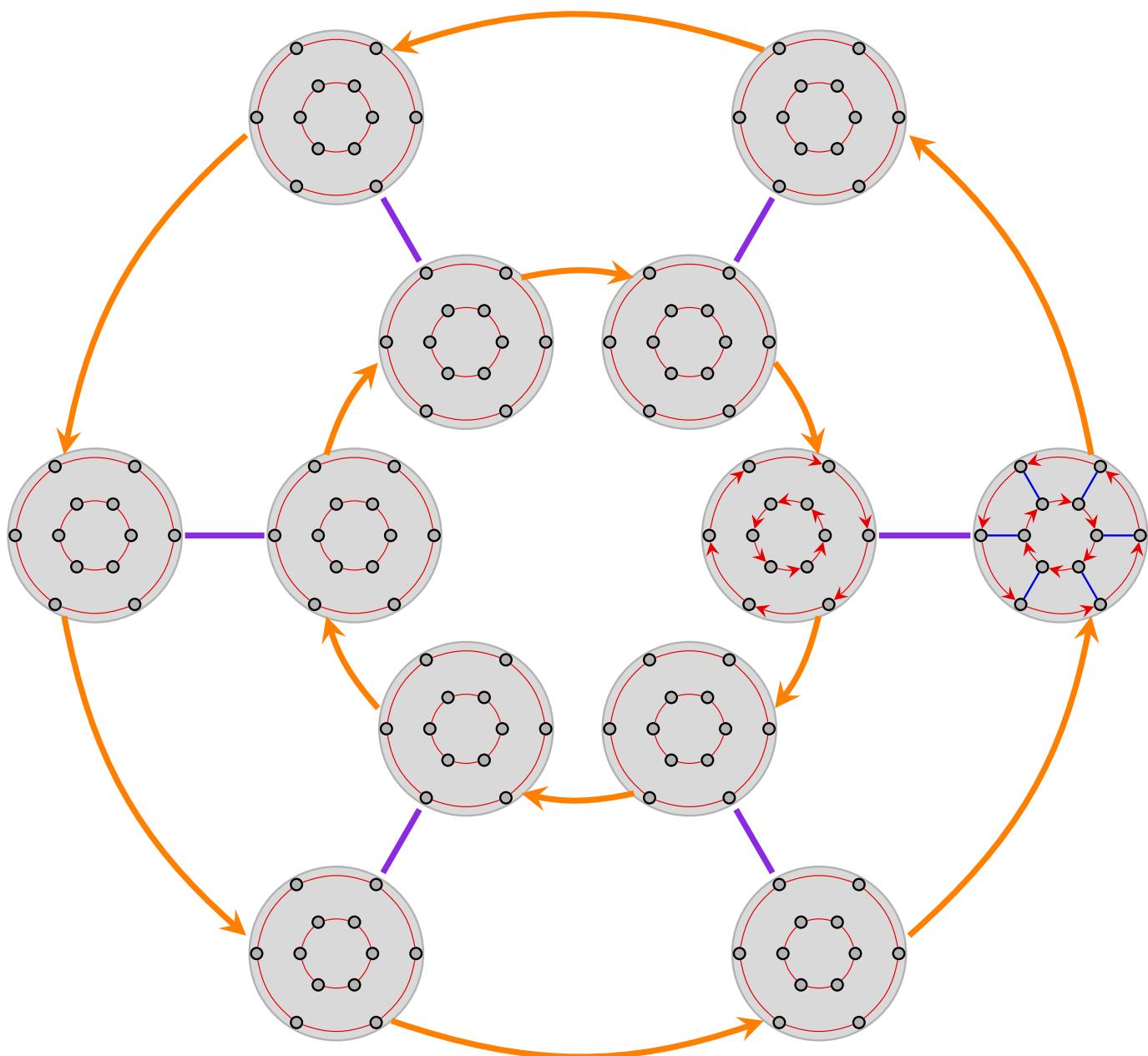


#1(b): Fixed point table of D_6 acting on its subgroups by conjugation.

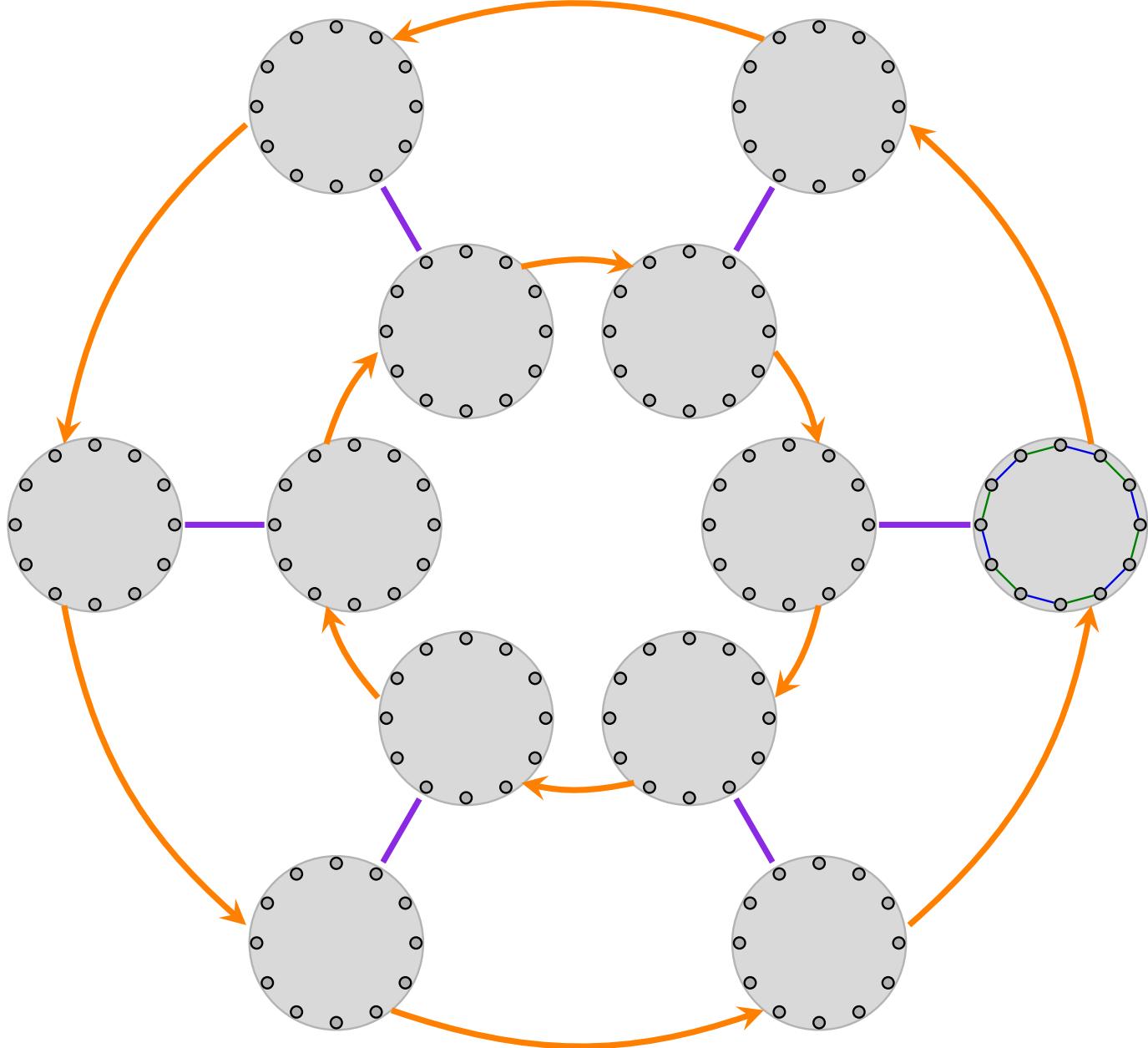
#1(c): The action graph poset of $D_6 = \langle r, f \rangle$, constructed by collapsing its Cayley graph by right cosets of its subgroups.



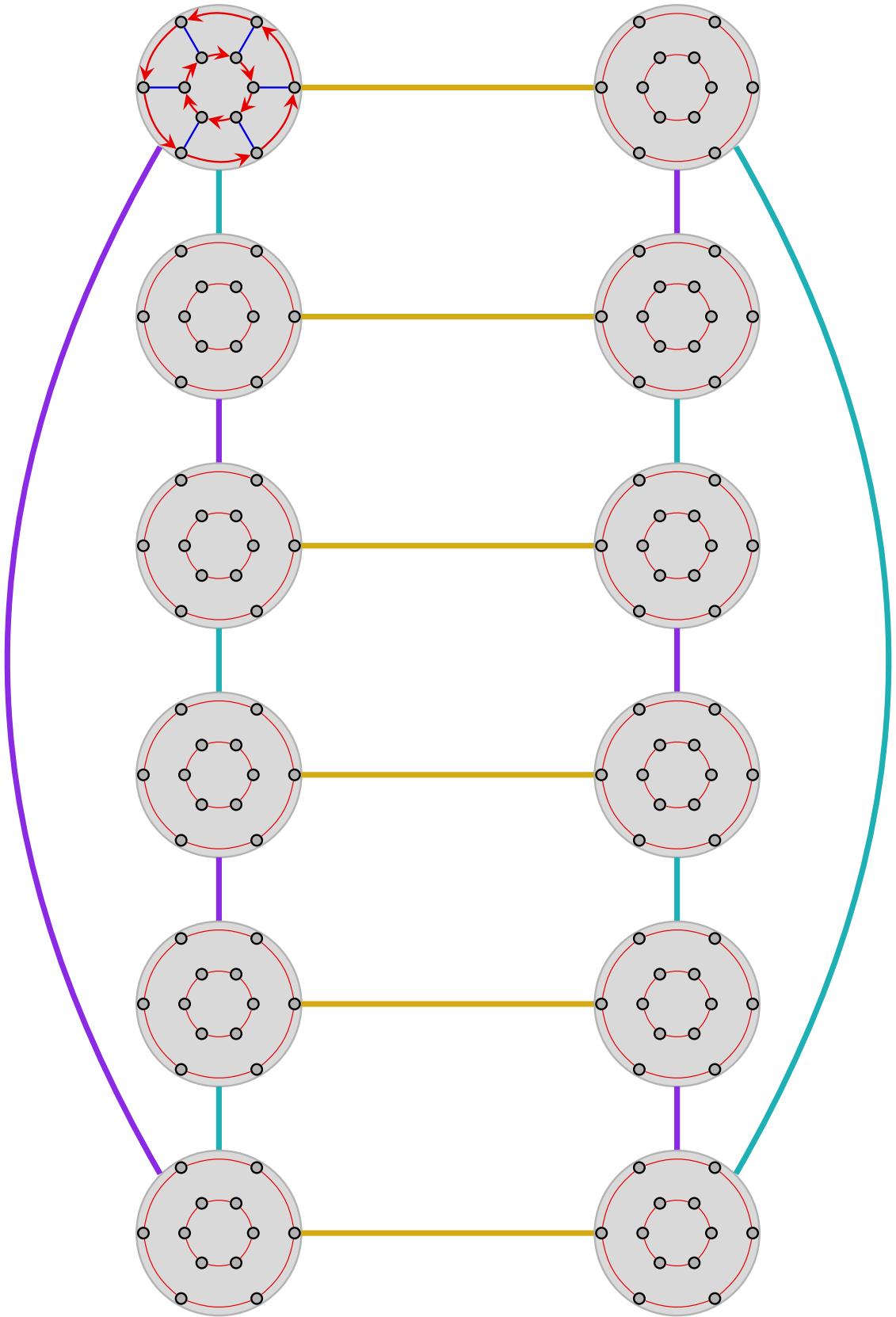
#1(d): Cayley graph of the automorphism group $\text{Aut}(D_6) \cong D_6$, with the nodes labeled by rewired copies of the Cayley graph of $D_6 = \langle r, f \rangle$, and with the inner automorphisms labeled.



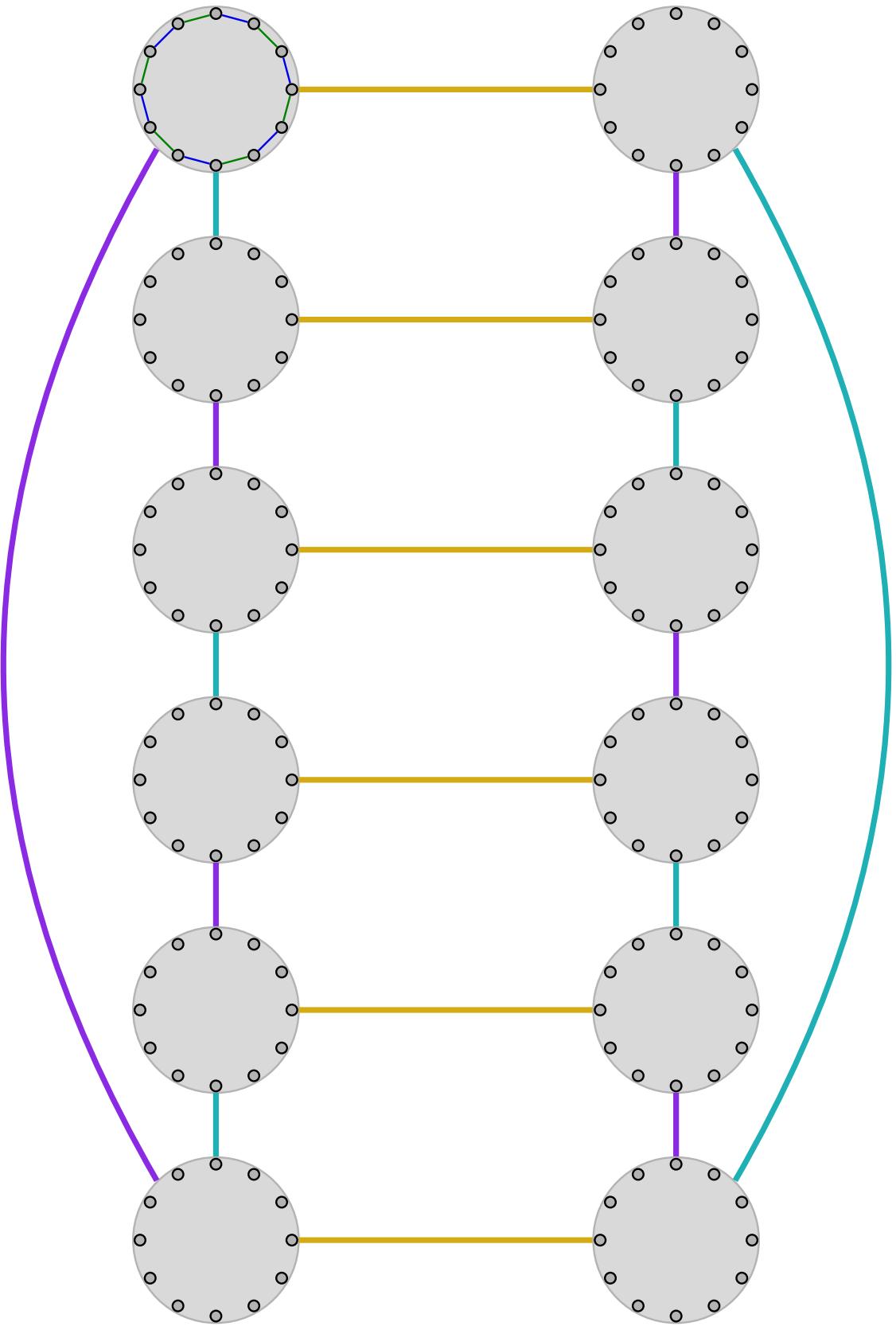
#1(d): Cayley graph of the automorphism group $\text{Aut}(D_6) \cong D_6$, with the nodes labeled by rewired copies of the Cayley graph of $D_6 = \langle s, t \rangle = \langle f, rf \rangle$, and with the inner automorphisms labeled.



#1(d): Cayley graph of the automorphism group $\text{Aut}(D_6) \cong D_3 \rtimes C_2$.



#1(d): Cayley graph of the automorphism group $\text{Aut}(D_6) \cong D_3 \rtimes C_2$.



#1(e): Partition of $\text{Aut}(D_6) \cong \text{Inn}(D_6) \rtimes \langle \eta \rangle$ into cosets of $\text{Inn}(D_6)$.

$$\text{Inn}(D_6) = \langle \varphi_r, \varphi_f \rangle$$

Id	\bigcap_1	\bigcap_r	\bigcap_{r^2}	\bigcap_f	\bigcap_{r^2f}	\bigcap_{r^4f}
	r^3	r^5	r^4	rf	r^3f	r^5f

$$\mathrm{Inn}(D_6)\omega$$

 1	r	r^2	f	$r^2 f$	$r^4 f$
 r^3	r^5	r^4	$r f$	$r^3 f$	$r^5 f$

ω

φ_r

$\textcircled{1}$	r	r^2	f	r^2f	r^4f
$\textcircled{2}$	r^3	r^5	r^4	rf	r^3f

\textcirclearrowleft 1	r	r^2	f	$r^2 f$	$r^4 f$
\textcirclearrowleft r^3	r^5	r^4	$r f$	$r^3 f$	$r^5 f$

φ_rω

$$\varphi_{r^2}$$

$\begin{smallmatrix} \curvearrowleft \\ 1 \end{smallmatrix}$	r	r^2	f	r^2f	r^4f
$\begin{smallmatrix} \curvearrowleft \\ r^3 \end{smallmatrix}$	r^5	r^4	rf	r^3f	r^5f

	r	r^2	f	$r^2 f$	$r^4 f$	
	r^3	r^5	r^4	$r f$	$r^3 f$	$r^5 f$

$$\varphi_r \omega$$

$$\varphi_f$$

r^0	1	r	r^2	f	$r^2 f$	$r^4 f$
r^3		r^5	r^4	$r f$	$r^3 f$	$r^5 f$

\textcirclearrowleft 1	r	r^2	f	$r^2 f$	$r^4 f$
\textcirclearrowleft r^3	r^5	r^4	$r f$	$r^3 f$	$r^5 f$

$$\varphi_f \omega$$

φrf

$\begin{smallmatrix} \curvearrowleft \\ 1 \end{smallmatrix}$	r	r^2	f	r^2f	r^4f
$\begin{smallmatrix} \curvearrowleft \\ r^3 \end{smallmatrix}$	r^5	r^4	rf	r^3f	r^5f

 1	r	r^2	f	$r^2 f$	$r^4 f$
 r^3	r^5	r^4	$r f$	$r^3 f$	$r^5 f$

$\varphi_{rf\omega}$

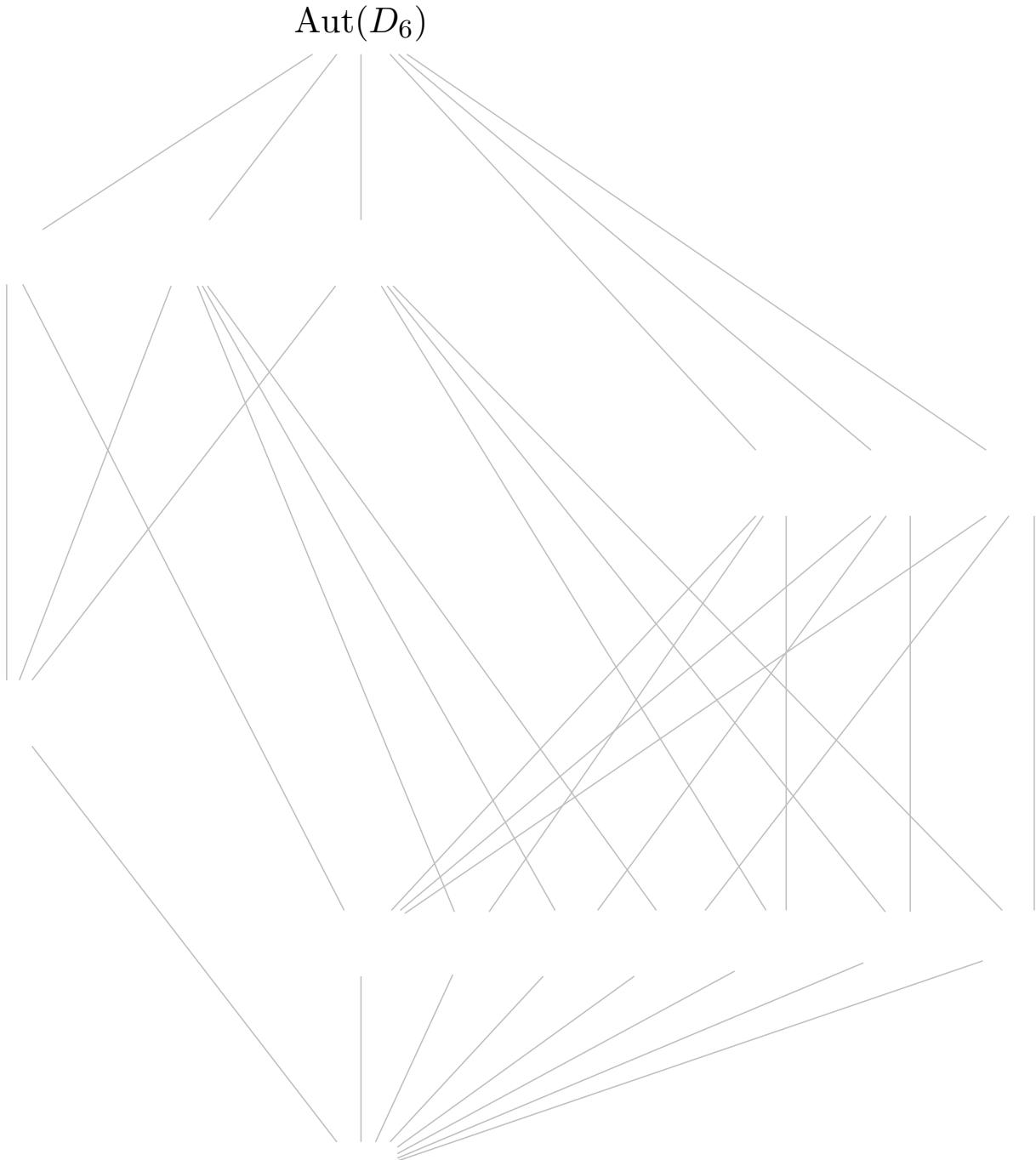
$$\varphi_{r^2f}$$

\curvearrowleft 1	r	r^2	f	r^2f	r^4f
\curvearrowleft r^3	r^5	r^4	rf	r^3f	r^5f

 1	r	r^2	f	$r^2 f$	$r^4 f$
 r^3	r^5	r^4	$r f$	$r^3 f$	$r^5 f$

$$\varphi_{r^2f}\omega$$

#1(f): Subgroup lattice of $\text{Aut}(D_6) \cong \text{Inn}(D_4) \rtimes \langle \eta \rangle = \langle \varphi_r, \varphi_f, \eta \rangle$.



#1(h): Action graph and fixed point table of the action of $\text{Aut}(D_6) = \langle \varphi_r, \varphi_f, \eta \rangle \cong \text{Inn}(D_6) \rtimes \langle \eta \rangle$ on D_6 .

1	r	r^2	f	r^2f	r^4f
r^3	r^5	r^4	rf	r^3f	r^5f

#1(h): Action graph and fixed point table of the action of $\text{Aut}(D_6) = \langle \varphi_r, \varphi_f, \omega \rangle$ on D_6 , where $\omega \in \text{Aut}(D_6)$ is the outer automorphism $f \mapsto rf$ of order 6, that cyclically rotates the axes of reflection.

1	r	r^2	f	r^2f	r^4f
r^3	r^5	r^4	rf	r^3f	r^5f

#1(i): Action graph and fixed point table of the action of $\text{Aut}(D_6) = \langle \varphi_r, \varphi_f, \eta \rangle \cong \text{Inn}(D_6) \rtimes \langle \eta \rangle$ on the conjugacy classes of D_6 .



	cl(1)	cl(r^3)	cl(r)	cl(r^2)	cl(f)	cl(rf)
Id						
φ_r						
φ_{r^2}						
φ_f						
φ_{rf}						
φ_{r^2f}						
η						
$\varphi_r\eta$						
$\varphi_{r^2}\eta$						
$\varphi_f\eta$						
$\varphi_{rf}\eta$						
$\varphi_{r^2f}\eta$						

#1(i): Action graph and fixed point table of the action of $\text{Aut}(D_6) = \langle \varphi_r, \varphi_f, \omega \rangle$ on the conjugacy classes of D_6 , where $\omega \in \text{Aut}(D_6)$ is the outer automorphism $f \mapsto rf$ of order 6, that cyclically rotates the axes of reflection.

$\text{cl}(1)$

$\text{cl}(r^3)$

$\text{cl}(r)$

$\text{cl}(r^2)$

$\text{cl}(f)$

$\text{cl}(rf)$

	$\text{cl}(1)$	$\text{cl}(r^3)$	$\text{cl}(r)$	$\text{cl}(r^2)$	$\text{cl}(f)$	$\text{cl}(rf)$
Id						
φ_r						
φ_{r^2}						
φ_f						
φ_{rf}						
φ_{r^2f}						
ω						
$\varphi_r\omega$						
$\varphi_{r^2}\omega$						
$\varphi_f\omega$						
$\varphi_{rf}\omega$						
$\varphi_{r^2f}\omega$						