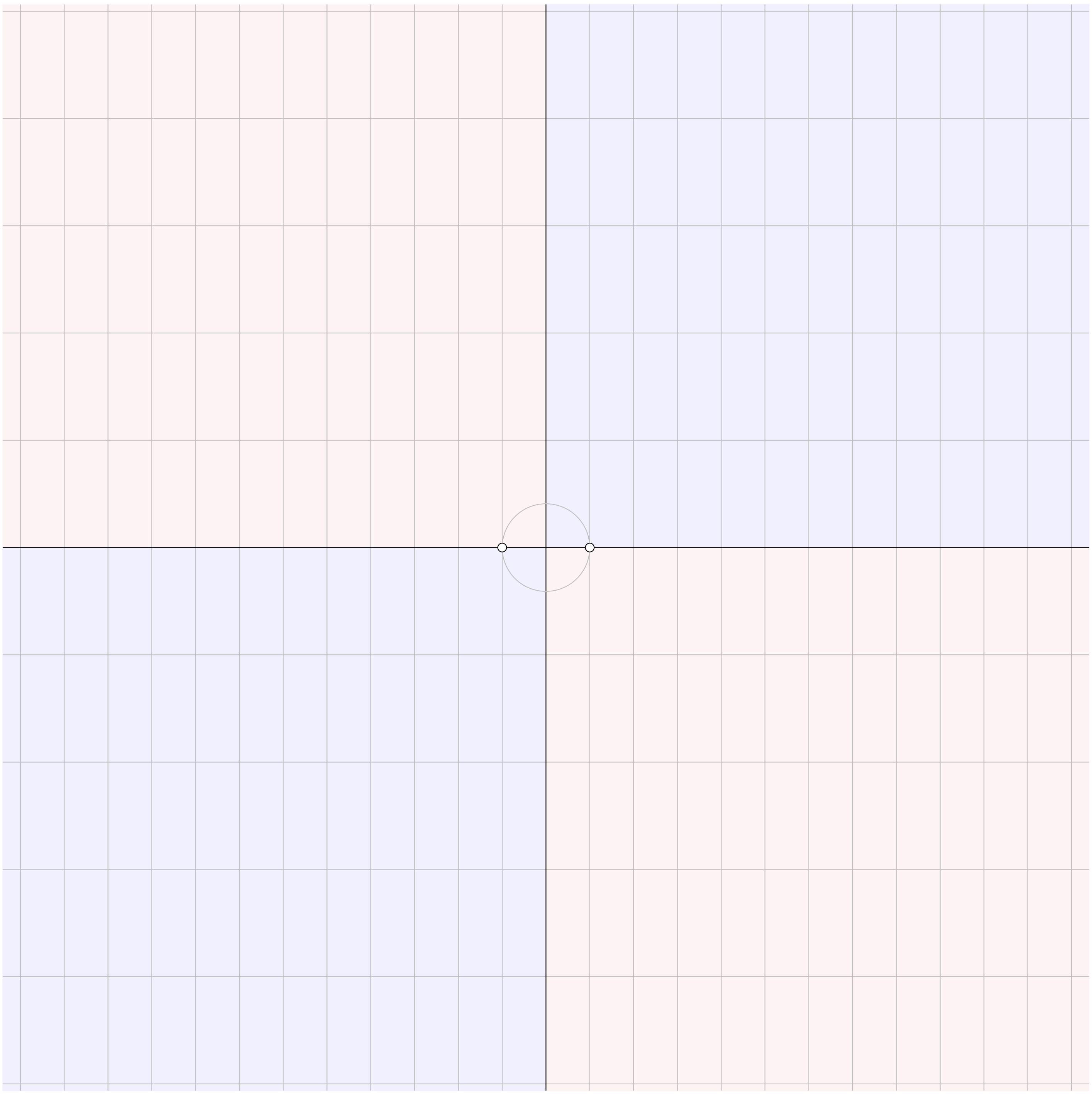
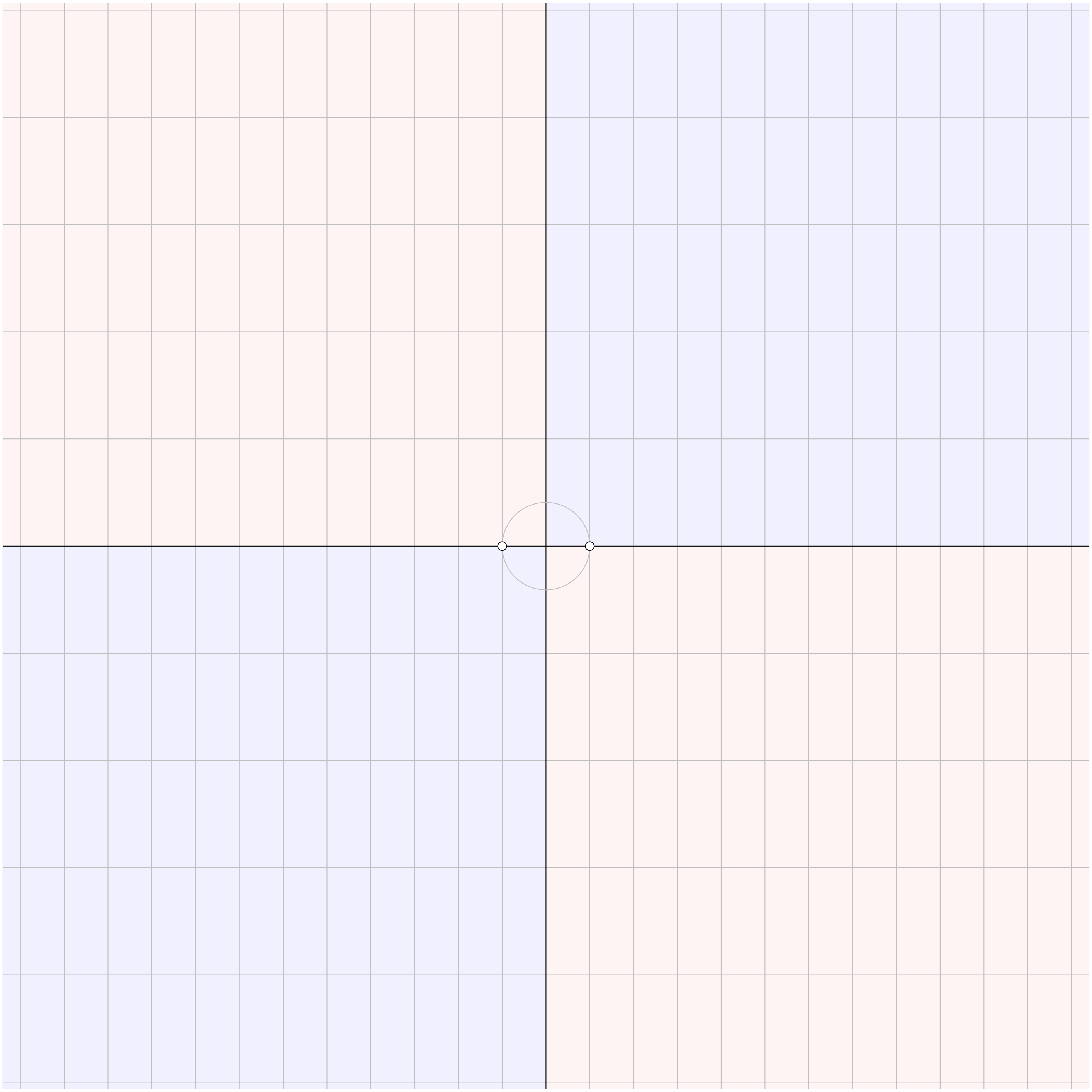
Supplemental material for Math 8510, HW 13

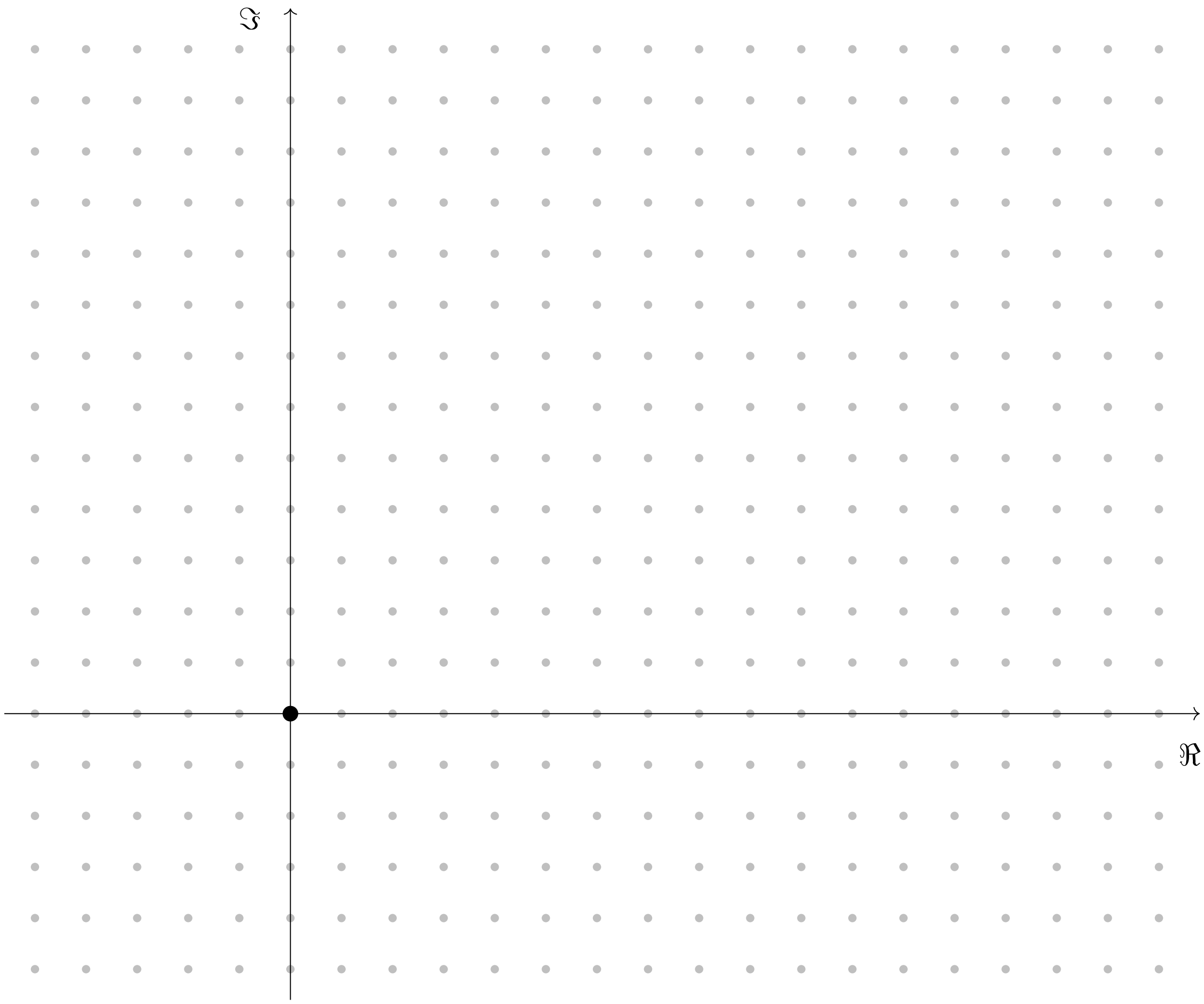
#1: The norms of some quadradic integers in the ring $R_{-6} = \mathbb{Z}[\sqrt{-6}] = \left\{a + b\sqrt{-6} \mid a, b \in \mathbb{Z}\right\}$.



#1: Some **primes** and **non-prime irreducibles** in the ring $R_{-6} = \mathbb{Z}[\sqrt{-6}] = \{a + b\sqrt{-6} \mid a, b \in \mathbb{Z}\}$ of quadratic integers. Ones with a ramified factor are in purple.

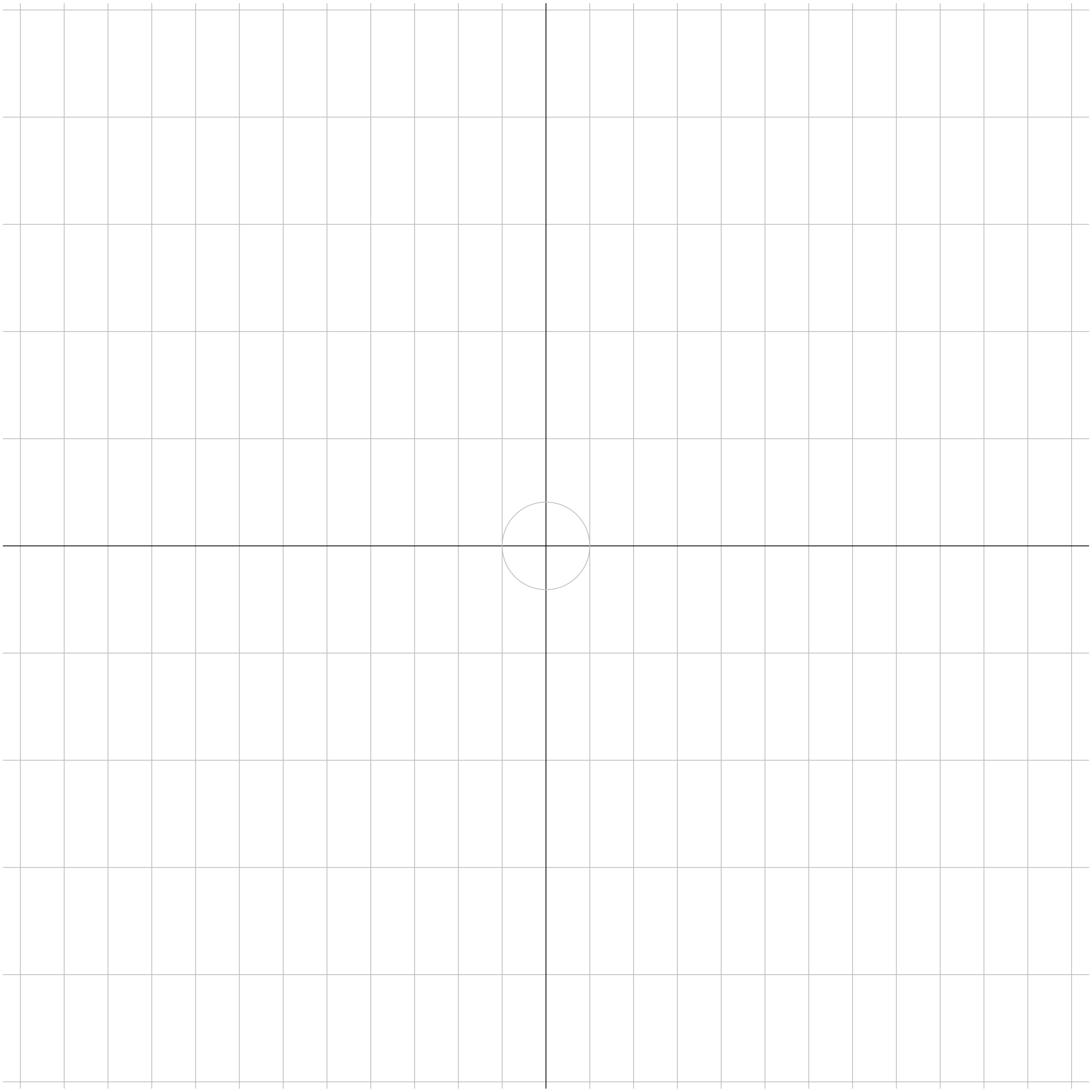


#5(a): The division algorithm in the Gaussian integers $R_{-1} = \mathbb{Z}[i] = \{a+bi \mid a, b \in \mathbb{Z}\}$ showing the possible quotients q and remainders r, upon dividing $b = 3+i$ into $a = 9+8i$



#5(b): An explicit example of $a, b \in R_{-6} = \mathbb{Z}[\sqrt{-6}] = \{a + b\sqrt{-6} \mid a, b \in \mathbb{Z}\}$ for which there are no quotient q and remainder r satisfying

$$a = bq + r, \qquad 0 \leq N(r) < N(b).$$



#5(b) (alternate grid): An explicit example of $a, b \in R_{-6} = \mathbb{Z}[\sqrt{-6}] = \{a + b\sqrt{-6} \mid a, b \in \mathbb{Z}\}$ for which there are no quotient q and remainder r satisfying

$$a = bq + r, \qquad 0 \leq N(r) < N(b).$$