Read: Lax, Chapter 3, pages 25–28, and Chapter 4, pages 32–43.

1. Show that whenever meaningful,

   (a) $(ST)' = T'S'$

   (b) $(T+R)' = T' + R'$

   (c) $(T^{-1})' = (T')^{-1}$.

   Here, S' denotes the transpose of S. Carefully describe what you mean by "whenever meaningful" in each case.

2. Both of the equalities in this problem following easily from $N_{T'} = R_T^{\perp}$, either by applying it to the transpose map, or taking the annihilator of both sides of this equation. Give an alternate direct algebraic proof of each, using the definitions. As usual, you should canonically identity X'' with X.

   (a) $N_T = R_{T'}^{\perp}$

   (b) $N_{T'}^{\perp} = R_T$.

3. Let $\mathcal{P}_n$ be the vector space of all polynomials over $\mathbb{R}$ of degree less than n.

   (a) Show that the map $T\colon \mathcal{P}_3 \to \mathcal{P}_4$ given by

   $$T(p(x)) = 6\int_1^x p(t)\,dt$$

   is linear. Determine whether it is 1–1 or onto.

   (b) Let $\mathcal{B}_3 = \{1, x, x^2\}$ be a basis for $\mathcal{P}_3$ and let $\mathcal{B}_4 = \{1, x, x^2, x^3\}$ be a basis for $\mathcal{P}_4$. Find the matrix representation of T with respect to these bases.

4. Let $T\colon X \to U$, with $\dim X = n$ and $\dim U = m$. Show how to construct bases $\mathcal{B}_X$ for X and $\mathcal{B}_U$ for U such that the matrix of T in block form is

   $$M = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$$

   where I_r is the $r \times r$ identity matrix, and the other blocks are either empty or contain all zeros.

5. Let X be a vector space with basis x_1, x_2, x_3, and consider the linear map $T : X \to X$ with matrix representation $\begin{bmatrix} 1 & -1 & 0 \\ 0 & 2 & -2 \\ -3 & 0 & 3 \end{bmatrix}$ with respect to this basis. What is the matrix representation of T with respect to the basis $x_1 - x_2$, $x_2 - x_3$, $x_1 + x_3$.