Lecture 5.8: Sequences and convergence

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Sequences of real and complex numbers

Definition

A sequence $\{a_k\}$ of numbers:

1. converges to a limit a if $|a_k - a| \to 0$. We write $\lim_{k\to\infty} a_k = a$.
2. is Cauchy if $|a_k - a_j| \to 0$ as $j, k \to \infty$.
3. is bounded if for some $R \geq 0$, every $|a_k| < R$.

The real (and complex) numbers are complete: every Cauchy sequence converges.

They are also locally compact: every bounded sequence contains a convergent subsequence.

Goal

Extend these properties from numbers to finite-dimensional inner product spaces.

Sequences of vectors

Definition

A sequence $\{x_k\}$ of vectors:

1. converges to a limit x if $||x_k - x|| \to 0$. We write $\lim_{k\to\infty} x_k = x$.
2. is Cauchy if $||x_k - x_j|| \to 0$ as $j, k \to \infty$.
3. is bounded if for some $R \geq 0$, every $||x_k|| < R$.

Completeness of inner product spaces

Proposition 5.17

Every finite-dimensional inner product space is complete.

Local compactness of inner product spaces

Proposition 5.18

Let X be an inner product space. Then X is locally compact if and only if $\dim X < \infty$.