$f_1 = (x_1 \lor x_{21}) \land \overline{x_{22}}$  $f_2 = 0$ 

# MATH 9850: ALGEBRAIC SYSTEMS BIOLOGY

FALL 2024: MWF 1:25-2:15

DR. MATTHEW MACAULEY

## What is algebraic biology?

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 $x_2 = I - N -$ 

 $X_{\Lambda} = I \vdash N$ -

It may come as a surprise to mathematicians and biologists alike when they first hear the term "Algebraic Biology." The reaction may be that of inquisitive curiosity, skepticism, or cynicism, as mathematicians have earned a reputation for occasionally making questionable abstractions and constructing frameworks that are perhaps too detached from reality to be more than just amusement. Another reason is that abstract algebra is almost never taught in biology or biomathematics courses, nor are applications to biology usually presented in algebra courses. However, *linear algebra* undeniably plays a fundamental role in applied fields such as mathematical biology. Systems of linear polynomials arise both as models of natural phenomena and as approximations of nonlinear models. Similarly, it should not hard be to surmise that systems of *nonlinear polynomials* can also arise from biological problems. A natural setting to work with multivariate polynomials are commutative rings, and the branch of mathematics that involves solving systems of such polynomials is *algebraic geometry*. Polynomials arise in models of biological systems across a variety of frameworks, from classical differential equations, to Boolean networks, to statistical models in phylogenetics and genomics, to topics in neuroscience. Broadly speaking, *algebraic biology* is an umbrella term that encompasses a wide range of problems from mathematical biology where polynomials arise.

The primary focus of this course will be on discrete models of biological systems. If the state space is binary, these are sometimes called *Boolean networks*. In general, any function over a finite field is a multvariate polynomial, which means that discrete models of this type can viewed as *algebraic models*, and analyzed using tools from computational algebra. The discrete framework is an alternative to more classical modeling techniques involving differential equations. Both have their advantages and disadvantages, and we will learn about both of these. Additionally, it cannot be understated how many new mathematical research problems, that are interesting on their own right, arose from the original application of algebra to biology.

19

#### Course Audience

This course is targeted to graduate students or advanced undergraduates in mathematics, but it will be accessible to students in other quantitative fields, such as systems biology or bioengineering, provided that they are familiar with ideas from linear algebra and differential equations. Though ideals will arise from commutative and computational algebraic geometry, no advanced knowledge of these areas is required; only the mathematical maturity to pick up the basics if needed. Especially in the second half of the class, there is some flexible about which topics we cover, which can depend on the interests of the class. No biological background will be assumed.

## Course Plan

This is a topics course, and will be much less work-intensive than a regular graduate prelim course. There will be light homework, to make sure that students keep up with the material beyond just attending lecture. Even in a topics class, there is no substitute for problem solving. Students will also complete a project, consisting of a short write-up and presentation of a research paper and/or topic related to algebraic biology. This will allow students to explore topics that they might be interested in, which we are not covering in lecture.

#### Questions Contact: macaule@clemson.edu, O-325 Martin Hall.

### More info

The case for algebraic biology: from research to education | link

 $X_{10} \vee X_{22}$ 

- Computer algebra in systems biology | link
- Can biology lead to new theorems? | link

Algebraic models, inverse problems, and pseudomonomials from biology | link