1. **Reduction**. Consider the following Boolean network model of the *lac* operon.

$$
\begin{aligned}
f_1 &= x_4 \wedge \overline{x_5} \wedge \overline{x_6} & f_8 &= x_9 \vee x_{10} \\
f_2 &= x_1 & f_9 &= x_3 \wedge \overline{G_e} \wedge L_e \\
f_3 &= x_1 & f_{10} &= \overline{G_e} \wedge \big(L_e \vee (x_3 \wedge L_{em})\big) \\
f_4 &= \overline{G_e} & f_{G_e} &= G_e \\
f_5 &= \overline{x_7} \wedge \overline{x_8} & f_{L_e} &= L_e \\
f_6 &= x_5 \vee (\overline{x_7} \wedge \overline{x_8}) & f_{L_{em}} &= L_{em} \\
f_7 &= x_2 \wedge x_9 & &
\end{aligned}
$$

Here, the variables represent

$$(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}) = (M, B, P, C, R, R_m, A, A_m, L, L_m).$$

(a) Use Macaulay2 to reduce this Boolean network, as much as possible.

(b) Draw the wiring diagram of the reduced network. Find its fixed point(s) and use these to determine the fixed point(s) of the original network by back-substitution.