Class schedule: Math 4110, Fall 2025

- **Week 1: 8/20–8/22**. We spent Wednesday doing introductions, and an overview of the class. We briefly discussed a history of combinatorics since the 20th century, which included Percy MacMahon, Paul Erdős, Gian-Carlo Rota, and Richard Stanley. On Friday, we gave a fun example of different ways to count a set of objects: (1) closed formula, (2) recurrence relation, (3) asymptotic formula, (4) generating function. Specifically, we counted the set of tilings of a $2 \times n$ board by dominoes, and discovered that the answer satisfies the Fibonacci recurrence: $t_n = t_{n-1} + t_{n-2}$. We used this to find the generating function, and got a formula in terms of the golden ratio.

- **Week 2: 8/25–8/29**. On Monday, we discussed the pigeonhole principle, and some fun appplications of it. On Wednesday, we reviewed proofs by induction. On Friday, we did strong induction, and an introduction to basic counting principles. If there are a ways to do Task A and b ways to do Task B, regardless of the outcome of Task B, then there are (i) ab ways to do Task A followed by Task B, and (ii) $a + b$ ways to do Task A or Task B. We saw an example involving the number of ways to properly color a map with q colors.

- **Week 3: 9/1–9/5**. No class Monday (labor day). We discussed strong induction and used it to prove a recurrence. Then we moved onto basic counting. We discussed how to count the number of ways to color a map with q colors, using q colors. Then we saw how to represent this with coloring graphs, and how every coloring represents an acyclic orientation. By relaxing the coloring condition, we can define labelings that are compatible with acyclic orientations, and from this we get a combinatorial reprocity formula: $\overline{\chi}(-q) = (-1)^n \chi(q)$, and plugging in $q = -1$ gives the number of acyclic orientations. Then we moved onto basic counting problems, involving permutations, subsets, and compositions. We saw how to use generating functions to represent these, and to prove new identities.

- **Week 4: 9/8–9/12**. On Monday, we covered multisets, a formula for them, and the multivariable generating function. On Wednesday, we discussed how to define the binomial coefficient $\binom{r}{k}$ for any real number r. This was motivated by the Taylor series of $(1 + x)^r$ at $x = 0$, and it gives us a combinatorial reciprocity formula for multisets: $\left(\!\binom{n}{k}\!\right) = (-1)^k \binom{-n}{k}$. We gave another version of the binomal theorem, but with two variables. Then we gave several combinatorial proofs: one involving Pascal's triangle, and Vandermode's identity. We spent Friday discussing Ehrhart theory, which counts the number of integer lattice points in a rational polytope. The central theorem is Erthart-Macdonal reciprocity, which says that $L_P(-n) = (-1)^k L_{P^\circ}(n)$. This relates the number of lattice points in a polytope to the number in its interior. This topic was motivated by the combinatorial reciprocity formula that $W(-n,k) = (-1)^{k-1}C(n,k)$, relating weak and strict partitions of n into k parts, and how this can be viewed geometrically.

- **Week 5: 9/15–9/19**. We started with multinomial coefficients, and the multinomial theorem. We derived the generating function for $(1-4x)^{-1/2}$, which involved the double factorial operation. On Wednesday, we moved onto the *twelvefold way*, which describes how to count functions $f\colon N \to K$ between finite sets. All of these can be thought of balls-in-bins problems. In particular, there are three restrictions on the functions (none, 1-1, onto), and four on whether the balls and/or bins are labeled. On Friday, we discussed set partitions, which are counted by the Stirling number of the second kind (if the number of parts are fixed), and the Bell numbers (if not).

- **Week 6: 9/22–9/26**. We began with partitions of a number n, and how to represent them with Ferrers diagrams. This allowed us to prove some basic properties. Then, we took a generating function approch, to prove a number of identities and results, such as that there are the same number of partitions of n into odd parts as there are in distinct parts. We also gave a bijective proof of this that utilizes the binary encoding of numbers. On Friday, we used probability generating functions to show that there is no way to weight two dice so that the probability distribution of their sum is uniform. Then, we switched topics to permutations, and gave basic definitions and notations, before proving a simple recurrence for the number of permutations of $[n]$ with k cycles.