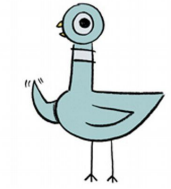**Topics**: Induction, the pigeonhole principle.

1. Without the use of AI or Google, use induction to prove that if $a_n = 1 + 2 + \cdots + n$, then $a_n = n(n+1)/2$. Then ask ChatGPT if your solution is correct, and report what it says. You can get full credit for doing this problem, even if your original solution was flawed.

2. Without the use of AI or Googling a solution, prove that for all positive integers n, the equality
$$1 + 2q + 3q^2 + \cdots + nq^{n-1} = \frac{1 - q^{n+1}}{(1-q)^2} - (n+1)\frac{q^n}{1-q}$$
holds for all complex numbers $q \neq 1$. [If you use AI or Google, please acknowledge it; you can still get full credit.]

3. Use ChatGPT to find surprising applications of the pigeonhole principal. In your own words, write up a complete solution. You will likely have to ask it multiple follow-up questions in order to get enough details to fully understand it. Include the prompts that you used.

4. Pick one of the exercises on Richard Stanley's worksheet on the pigenhole principle and solve it. This is from the Putnam Seminar at MIT, and you are encouraged to use Chat-GPT for help. Include any prompts that you use.