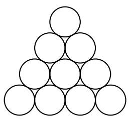
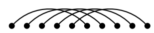
Topics: Catalan numbers

1. Everyone in class will be assigned a unique Catalan object from the following list.

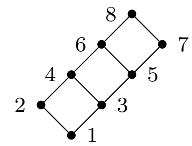


coin stacks; n in bottom row



non-nesting matchings of $[2n]$

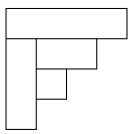
non-crossing matchings of $[2n]$



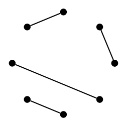
linear extensions of $\mathbf{2} \times \boldsymbol{n}$

1	2	5	7
3	4	6	

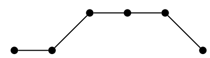
SYT of shape (n, n) or $(n, n-1)$



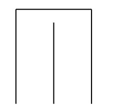
tilings of $(n, \dots, n)$ with n rectangles



n non-intersecting chords on $2n$ points



peakless Motzkin paths; n up & level steps



noncrossing Murasaki diagrams; n $\vert$'s

For your particular object, carry out the following steps.

(a) Use TikZ to make a visual of all $C_4 = 14$ examples for $n = 4$.

(b) Make a list of "natural" combinatorial statistics, and compute their distributions for $n = 4$.

(c) Construct a bijection to one of the Catalan objects that we saw in class.

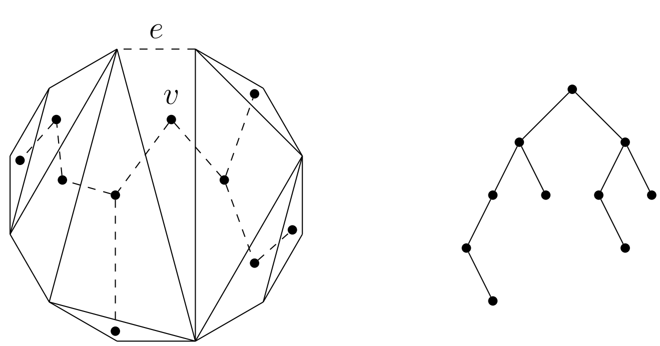
2. Richard Stanley maintains a list of over 200 combinatorial objects counted by the Catalan numbers. Over 160 of these are in his *Catalan Addendum* (`https://math.mit.edu/~rstan/ec/catadd.pdf`). Pick two of these, different from your classmates. At least one should be "visual" (not just numeric sequences), and along with your Catalan object from Problem #1, all three should seem distinctly different.

(a) Use TikZ to make a visual of all $C_4 = 14$ examples for $n = 4$.

(b) Give bijective proofs to show that these are all equivalent to each other. These should be self-contained; do not use other Catalan objects.

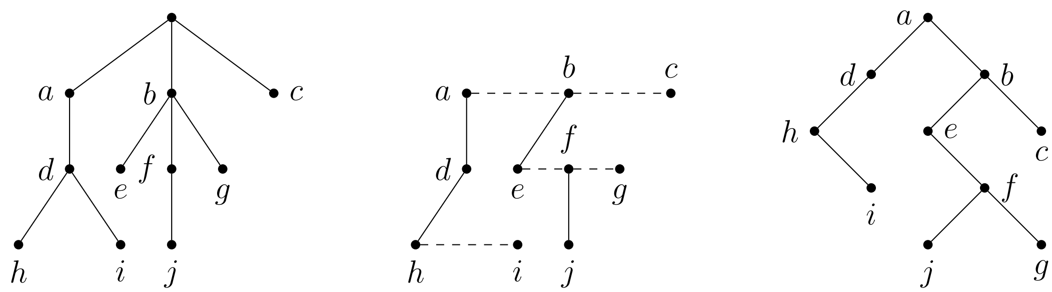
(c) Make a list of "natural" combinatorial statistics, and compute their distributions for $n = 4$. What is the interpretation of these under your bijections?

3. Consider the bijection we saw in class from the triangulations of an $(n + 2)$-gon, to the binary trees on n vertices; an example is shown below.



Construct the inverse of this map for the following binary trees.

4. Recall the bijection of de Bruijn and Marselt from plane trees on $n+1$ vertices to binary trees on n vertices; an example is shown below.



Construct the inverse of this bijection for the same two binary trees from the previous problem.