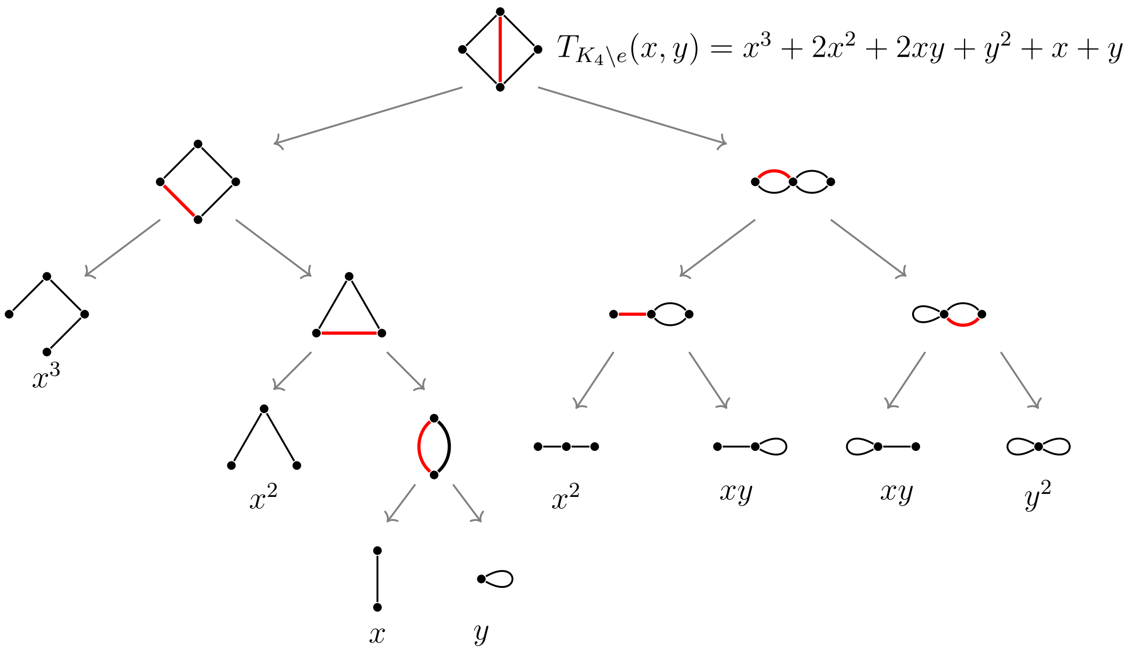
Topics: Tutte polynomial, Möbius functions

1. In class, we computed the Tutte polynomial recursively for the graph K_4 minus an edge, as follows:



   (a) Compute the Tutte polynomial $T_G(x,y)$, where G the "house graph" from HW 8.

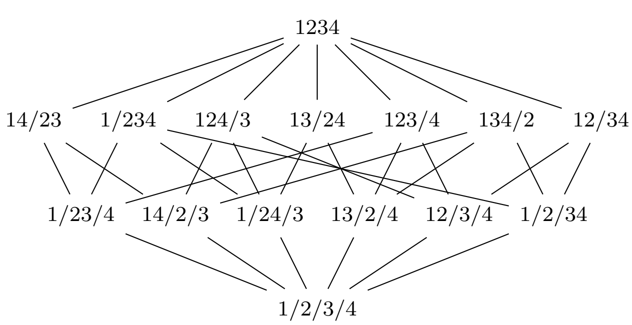
   (b) Find a graph G_1 such that $T_{G_1}(x,y) = xT_G(x,y)$.

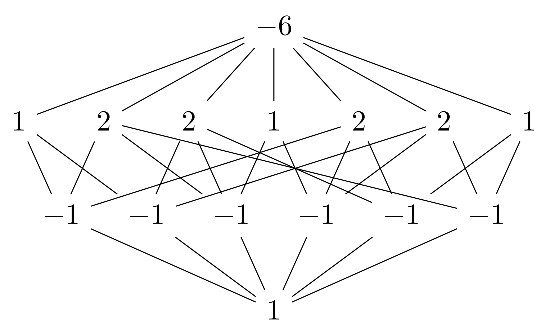
   (c) Find a graph G_2 such that $T_{G_2}(x,y) = yT_G(x,y)$.

2. The *Möbius function* of a poset is defined

$$\mu\colon \operatorname{Int}(P) \longrightarrow \mathbb{R}, \qquad \mu(x,y) = \begin{cases} 1 & x = y \\ -\sum_{z \leq x} \mu(z,x) & \text{otherwise.} \end{cases}$$

   A Hasse diagram of the partition lattice Π_4 is shown below, along with its nodes labeled by $\mu(\widehat{0},\pi)$, where $\widehat{0} = 1/2/3/4$ is the minimum element.

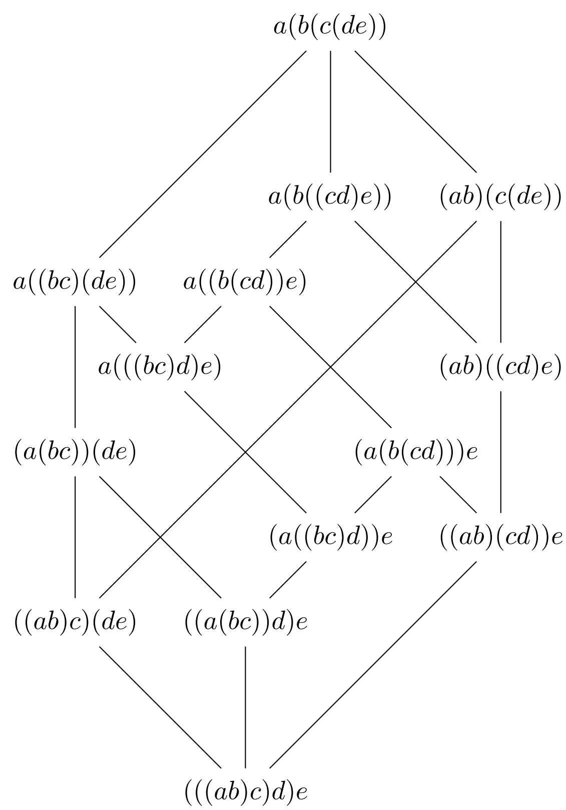




The Möbius function of Π_4 can be represented in matrix form, as follows.

1/2/3/4	1	−1	−1	−1	−1	−1	−1	1	2	2	1	2	2	1	−6
1/23/4	0	1	0	0	0	0	0	−1	−1	0	0	−1	0	0	2
14/2/3	0	0	1	0	0	0	0	−1	0	−1	0	0	−1	0	2
1/24/3	0	0	0	1	0	0	0	0	−1	−1	−1	0	0	0	2
13/2/4	0	0	0	0	1	0	0	0	0	0	−1	−1	−1	0	2
12/3/4	0	0	0	0	0	1	0	0	0	−1	0	−1	0	−1	2
1/2/34	0	0	0	0	0	0	1	0	−1	0	0	0	−1	−1	2
14/23	0	0	0	0	0	0	0	1	0	0	0	0	0	0	−1
1/234	0	0	0	0	0	0	0	0	1	0	0	0	0	0	−1
124/3	0	0	0	0	0	0	0	0	0	1	0	0	0	0	−1
13/24	0	0	0	0	0	0	0	0	0	0	1	0	0	0	−1
123/4	0	0	0	0	0	0	0	0	0	0	0	1	0	0	−1
134/2	0	0	0	0	0	0	0	0	0	0	0	0	1	0	−1
12/34	0	0	0	0	0	0	0	0	0	0	0	0	0	1	−1
1234	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Create an analogous diagram and matrix for the Möbius function of the Tamari lattice.



3. Let $\mathcal{F}_n$ be the set of labeled forests on $[n]$. To every forest $F \in \mathcal{F}_n$, we can associate a partition $\pi \in \Pi_n$ defined by the connected components; write this as $c(F) = \pi$. Set up a Möbius inversion problem by defining two functions $f, g \colon \Pi_n \to \mathbb{Z}$, where an evaluation of one of them gives Cayley's formula for the number of labeled trees. Explain your steps, but you do *not* need to solve it.