Weekly schedule: Math 4120, Fall 2026

- **WEEK 1: 8/19–8/21**. Course overview on Wednesday. We also convered the Chapter 1 slides (pp. 1–8, 14–22). The corresponding YouTube lectures are:
  - Lecture 1.1, Groups, symmetry, and Cayley graphs (0:00–24:59)
  - Lecture 1.2: Groups from puzzles

  Quiz 1 next Monday.

  Summary & key ideas. We introduced *Cayley graphs*, and saw several examples of groups: the symmetries of a rectangle, and of a triangle. These define algebraic *relations*. The same group can have very different looking Cayley graphs depending on generating sets. We briefly discussed puzzles like the Rubik's cub, mostly just for fun, but there is some deep group theory involved in that.

  To do:
  - Read over the slides we covered, formulate any questions you may have.
  - Look ahead at the remaining Chapter 1 slides for next week.

  Learn / memorize:
  - The three basic properties of a group (closure, identity, inverses)
  - Cayley graphs for $V_4 = \langle r, h \rangle$ and the "triangle symmetry graph" $D_3 = \langle r, f \rangle$ and $\langle s, t \rangle$.
  - Be able to use Cayley graphs to multiply elements, compute inverses, and find relations (two paths that correspond to the same element).

- **WEEK 2: 8/24–8/28**. Course overview on Tuesday. We also convered the Chapter 1 slides (pp. 9–13, 23–45) and the Chapter 2 slides (pp. 1–8). The corresponding YouTube lectures are:
  - Lecture 1.1, Groups, symmetry, and Cayley graphs (25:00–32:08)
  - Lecture 1.3, Group presentations
  - Lecture 1.4: Infinite symmetry groups
  - Lecture 1.5: Cayley tables
  - Lecture 1.6: The formal definition of a group
  - Lecture 2.1: Complex numbers and matrices
  - Lecture 2.2: Cyclic groups (0:00–1:50).

  Quiz 1 Wednesday. HW 1 due Monday.

  Summary & key ideas. We learned how to label Cayley graphs with actions. This motivated the idea of a *group presentation*, which encodes the Cayley graph by listing the *generators* and algebraic *relations*.

  We learned about infinite summetry group. One-dimensional symmetry groups are called *frieze groups*, and we classifed all 7 of them. The 2D analogue of these are were "17 different types of wallpaper", and the 3D analogue are the 230 "crystal groups." The quaternion group Q_8 was the first abstract group we've seen that doesn't the describe symmetries or actions. By constructing Cayley tables, we were able to see the concept of a *quotient*. We finally gave the formal definition of a group, and several examples of "things that look like groups but aren't", illustrating why a formal definition is needed.

  Moving into Chapter 2, we reviewed complex numbers, learned about roots of unity and how to factor $x^n - 1$ using cyclotomic polynomials.

To do: Read over the slides, formulate any questions you may have. *Familiarize yourself with the presentations of all of the groups we have seen.* Finish HW 1.

Learn / memorize:
- Be able to label nodes of a Cayley graph with group elements.
- Learn the presentations for groups we've seen (V_4, C_4, and Tri).
- How to multiply elements in Q_8.
- Presentations for groups we've seen (V_4, D_3, Q_8).
- Be able to differentiate various groups we've seen from their Cayley tables. One good way is to count the number of times the identity appears on the main diagonal (the # of $g \in G$ for which $g^2 = 1$).
- The formal definition of a group.
- The formal definition of a the order of a group, and the order of an element.
- The definition of the n^{th} roots of unity, and which ones are primitive.

- **WEEK 3: 8/31–9/4**. Two lectures covering the Chapter 2 slides (pp. 8–42). The corresponding YouTube lectures are
  - Lecture 2.2: Cyclic groups (1:51–33:26)
  - Lecture 2.3: Dihedral groups
  - Lecture 2.4: Abelian groups
  - Lecture 2.5: Groups of permutations (0:00–10:11)

  Quizzes 2 and 3 this week. HW 2 due next Monday.

Summary & key ideas.

We saw how to represent V_4 with matrices. We defined cyclic groups, both additively as $\mathbb{Z}_n = \langle 1 \rangle$ and multipliatively as $C_n = \langle r \rangle$. Finite cyclic groups describe rotations of an n-gon. The fully symmetry groups of n-gons are the *dihedral groups* D_n. We saw how to represent the group C_n and D_n with 2×2 matrices using roots of unity. There are infinite version of both of these: $C_\infty \cong \mathbb{Z}$ and D_∞ both occured as frieze groups.

We introduced the notion of a *cycle graph*, which contains all of the *maximal* cyclic subgroups in G. This gives a very different picture of the group vs. a Cayley graph, but both are useful. We saw how to construct the direct product of two groups, and prove that $\mathbb{Z}_n \times \mathbb{Z}_m \cong \mathbb{Z}_{nm}$ iff $\gcd(n, m) = 1$. We stated the big theorem that every finite (and finitely generated) abelian group is a direct product of cyclic groups. We saw two ways to classify these: by "prime powers", and "elementary divisors."

We introducted permutations, and several ways of encoding them, with *cycle notation* being our go-to method.

To do: Read over the slides, formulate any questions you may have. *Familiarize yourself with the presentations of all of the groups we have seen.* Finish HW 2.

Learn / memorize:
- When does $\langle k \rangle$ generate $\mathbb{Z}_k$ (or equivalently, $\langle r^k \rangle$ generate C_n).
- The cycle graphs for D_n and Q_8.
- How to construct a cycle graph given a group.
- How to construct Cayley graphs and presentations for D_n.
- How to represent C_n and D_n with 2×2 matrices over $\mathbb{C}$.
- That $\mathbb{Z}_n \times \mathbb{Z}_m \cong \mathbb{Z}_{nm}$ iff $\gcd(n, m) = 1$.
- *How to write a complete list of all abelian groups of some fixed order.* (I recommend the "prime power" classification.)
- Be able to list every group of order 4, 6, and 8, and draw their Cayley graphs.