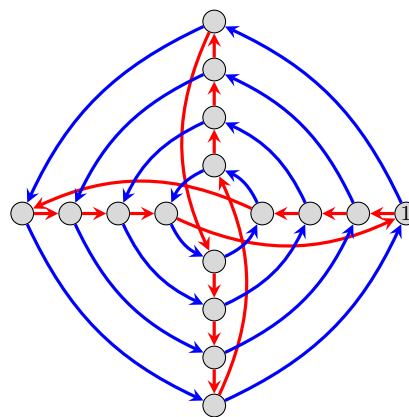
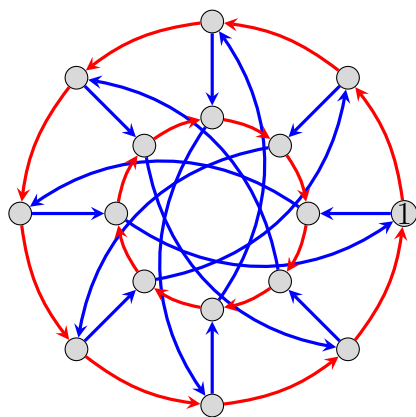


- For each n , sketch the n^{th} roots of unity on the unit circle, and list the primitive d^{th} roots for each $d \mid n$. Then factor $x^n - 1$ as a product of irreducible polynomials.
 - $n = 8$
 - $n = 9$
 - $n = 10$
 - $n = 16$.
- For each n from the previous problem, the set $U_n := \{k \mid 0 < k < n, \gcd(n, k) = 1\}$ forms a group under multiplication, where the result is taken modulo n . Construct a Cayley table, Cayley diagram, and determine to which familiar group it is isomorphic.
- Below are Cayley diagrams of the *generalized quaternion group* $Q_{16} = \langle \zeta_8, j \rangle$, defined by replacing $\zeta_4 = e^{2\pi i/4} = i$ with $\zeta_8 = e^{2\pi i/8} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$ in the quaternion group Q_8 .



- Draw these diagrams and label each node as $a + bi + cj + dk$. Then re-draw them with each node labeled as either $\pm\zeta^m$ or $\pm\zeta^m j$, where $\zeta = \zeta_8$ and $m = 0, 1, 2, 3$.
- Identifying elements of Q_{16} with their negatives defines a group on 8 elements:

$$\pm 1, \pm \zeta, \pm \zeta^2, \pm \zeta^3, \pm j, \pm \zeta j, \pm \zeta^2 j, \pm \zeta^3 j.$$

Construct a Cayley table and Cayley diagram. Which familiar group is this?

- For each part below, the two matrices given generate a group $G = \langle A, B \rangle$, where the binary operation is matrix multiplication. Draw a Cayley diagram for each group, write a presentation, and determine to which familiar group it is isomorphic.

$$(a) \quad A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}. \quad (c) \quad A = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}, B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}.$$

$$(b) \quad A = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}. \quad (d) \quad A = \begin{bmatrix} e^{2\pi i/8} & 0 \\ 0 & e^{-2\pi i/8} \end{bmatrix}, B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}.$$

- For the numbers below, list all abelian groups of that order by writing each one as a product of cyclic groups of prime power order. Then, determine which group it is isomorphic to of the form $\mathbb{Z}_{n_1} \times \cdots \times \mathbb{Z}_{n_k}$, where $n_{i+1} \mid n_i$.

$$(a) \quad 32 = 2^5 \quad (b) \quad 36 = 2^2 \cdot 3^2 \quad (c) \quad 400 = 2^4 \cdot 5^2 \quad (d) \quad p^3 q; \text{ primes } p \neq q$$