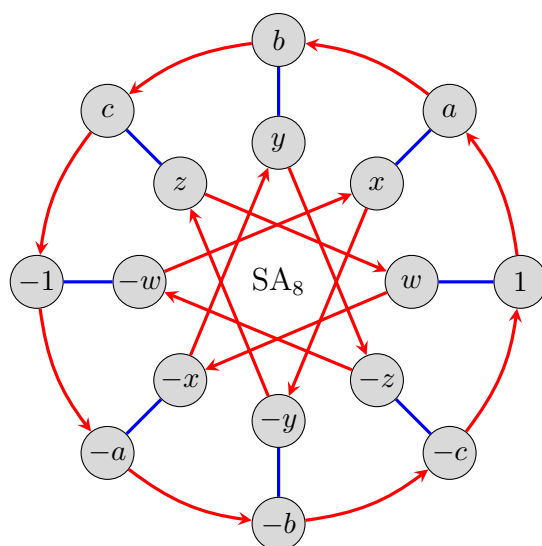
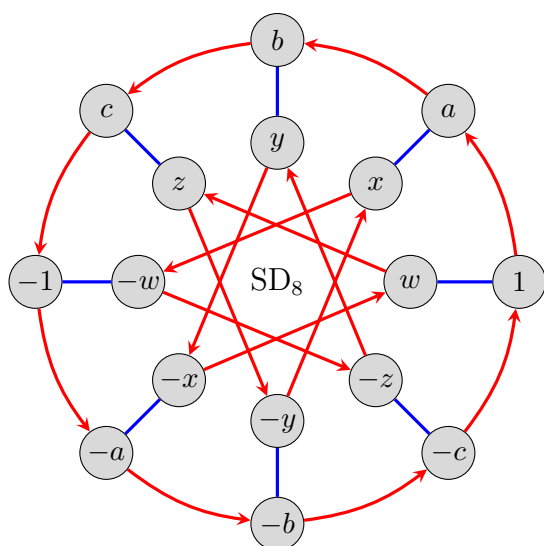
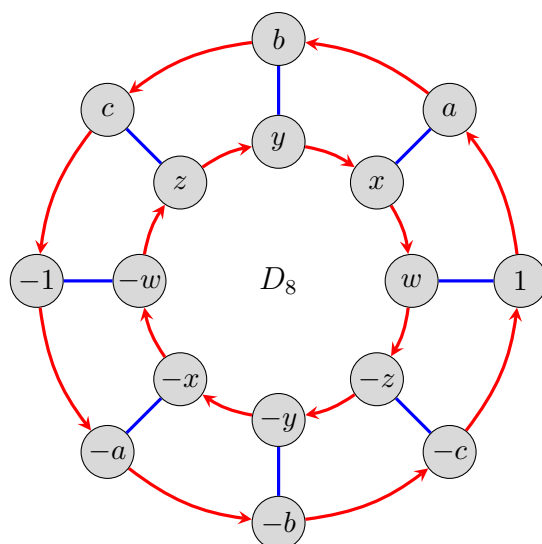
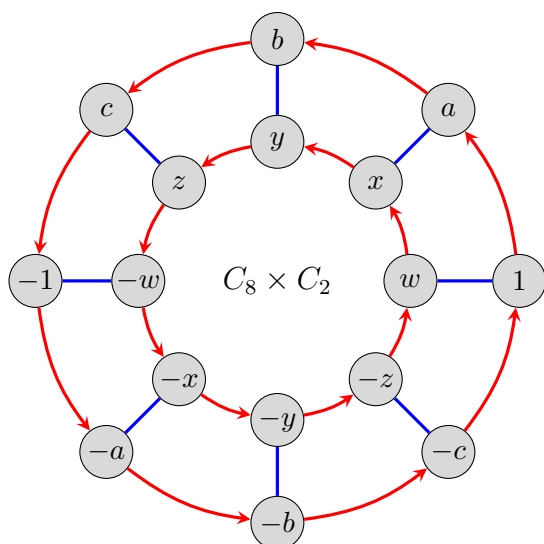


1. Cayley graphs for the four semidirect products of C_8 with C_2 are shown below, with a different labeling scheme on their nodes.



For all four of these groups, identifying each element with its “negative” yields a “quotient group” of order 8, like what we did with the dicyclic group $\text{Dic}_8 = Q_{16}$ in HW 2. Construct a Cayley table and Cayley graph for each of these quotient groups, using the elements

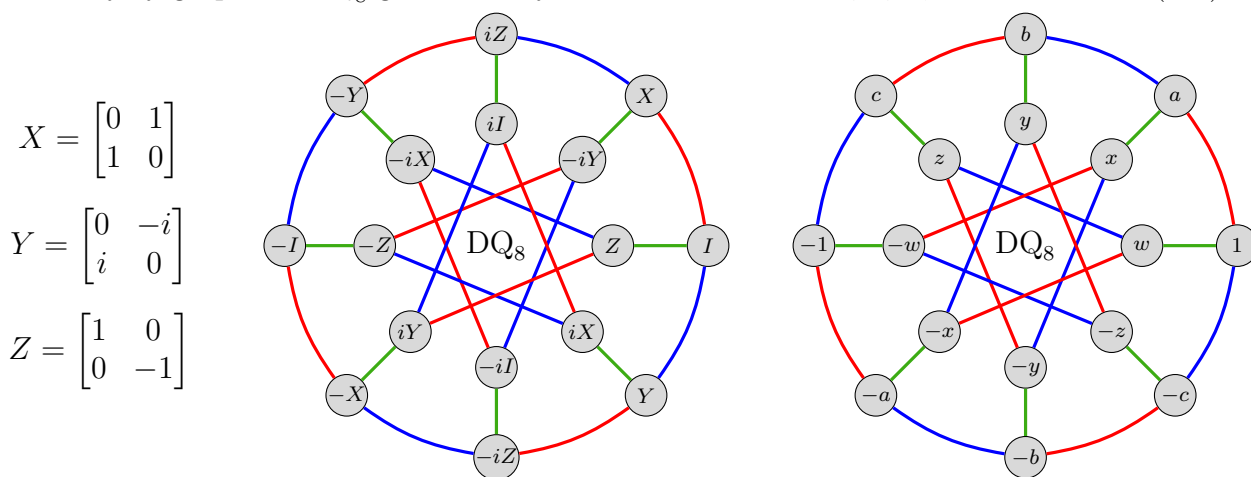
$$\pm 1, \pm a, \pm b, \pm c, \pm w, \pm x, \pm y, \pm z,$$

and determine to which familiar group each is isomorphic. If two groups give the same table and graph, you do not need to write this out twice.

2. The *diquaternion group* DQ_8 can be constructed from our standard representation of $Q_8 = \langle R_4, S, T \rangle$, along with the reflection matrix F in $D_n = \langle R_n, F \rangle$. That is,

$$DQ_8 \cong \langle i, j, k, f \rangle \cong \left\langle \underbrace{\begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}}_{R=R_4}, \underbrace{\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}}_S, \underbrace{\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}}_{T=RS}, \underbrace{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}_F \right\rangle.$$

A Cayley graph for DQ_8 generated by the *Pauli matrices* X, Y, Z , is shown below (left).



- (a) Find a presentation for $DQ_8 = \langle X, Y, Z \rangle$.
- (b) Construct a Cayley graph for $DQ_8 = \langle R, S, F \rangle$, and find a group presentation. Extra credit will be given for the best-looking construction.
- (c) Carry out the “quotient process” as was done for $C_8 \times C_2$, D_8 , SD_8 , and SA_8 in the previous problem; see the graph on the right.
3. For this problem, the use of an online matrix calculator, like <https://matrixcalc.org>, that can handle complex exponential inputs, is strongly recommended.

- (a) The dicyclic group Dic_n is only defined when n is even. However, if we try to define

$$Dic_3 := \left\langle \begin{bmatrix} \zeta_3 & 0 \\ 0 & \bar{\zeta}_3 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \right\rangle,$$

then the result is still a group. Determine which group this is, with justification.

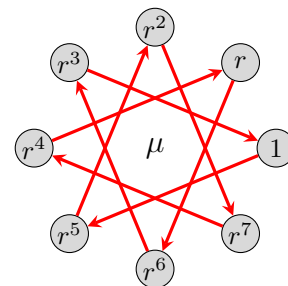
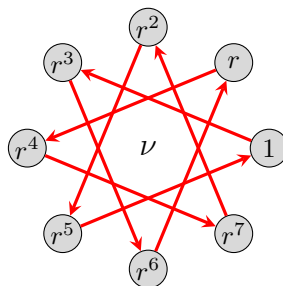
- (b) The diquaternion and semidihedral groups, DQ_n and SD_n , are only defined when $n = 2^m$. However, we can still define

$$DQ_6 := \left\langle \begin{bmatrix} \zeta_6 & 0 \\ 0 & \bar{\zeta}_6 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\rangle, \quad SD_3 := \left\langle \begin{bmatrix} \zeta_3 & 0 \\ 0 & -\bar{\zeta}_3 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\rangle.$$

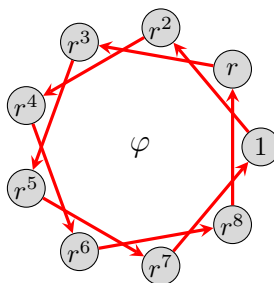
What groups are these? Construct a Cayley graph for each.

4. The automorphism group of C_n is isomorphic to U_n , the multiplicative group of integers modulo n , from HW 2, #2. For each of the following, construct a Cayley graph of $\text{Aut}(C_n)$ with the nodes labeled by re-wirings, and a Cayley table for this group. Then determine its isomorphism type.

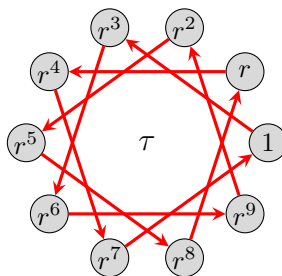
(a) $\text{Aut}(C_8) = \langle \nu, \mu \rangle$, defined by



(b) $\text{Aut}(C_9) = \langle \varphi \rangle$, defined by



(c) $\text{Aut}(C_{10}) = \langle \tau \rangle$, defined by



(d) $\text{Aut}(C_{16}) = \langle \rho, \sigma \rangle$, defined by

