1. Find the Laplace transform of the following functions by using a table of Laplace transforms
   (a) \( f(t) = -2 \)
   (b) \( f(t) = e^{-2t} \)
   (c) \( f(t) = \sin 3t \)
   (d) \( f(t) = te^{-3t} \)
   (e) \( f(t) = e^{2t} \cos 2t \)

2. Transform the given initial value problem into an algebraic equation involving \( Y(s) := \mathcal{L}(y) \), and solve for \( Y(s) \).
   (a) \( y'' + y = \sin 4t, \quad y(0) = 0, \quad y'(0) = 1 \)
   (b) \( y'' + y' + 2y = \cos 2t + \sin 3t, \quad y(0) = -1, \quad y'(0) = 1 \)
   (c) \( y' + y = e^{-t} \sin 3t, \quad y(0) = 0 \)

3. Find the inverse Laplace transform of the following functions.
   (a) \( Y(s) = \frac{2}{3 - 5s} \)
   (b) \( Y(s) = \frac{1}{s^2 + 4} \)
   (c) \( Y(s) = \frac{5s}{s^2 + 9} \)
   (d) \( Y(s) = \frac{3}{s^2} \)
   (e) \( Y(s) = \frac{3s + 2}{s^2 + 25} \)
   (f) \( Y(s) = \frac{2 - 5s}{s^2 + 9} \)
   (g) \( Y(s) = \frac{s}{(s + 2)^2 + 4} \)
   (h) \( Y(s) = \frac{3s + 2}{s^2 + 4s + 29} \)
   (i) \( Y(s) = \frac{2s - 2}{(s - 4)(s + 2)} \)
   (j) \( Y(s) = \frac{3s^2 + s + 1}{(s - 2)(s^2 + 1)} \)

4. Use the Laplace transform to solve the following initial value problems.
   (a) \( y' - 4y = e^{-2t}t^2, \quad y(0) = 1 \)
   (b) \( y'' - 9y = -2e^t, \quad y(0) = 0, \quad y'(0) = 1 \)
5. Find the Laplace transform of the given functions.
   (a) \(3H(t - 2)\)
   (b) \((t - 2)H(t - 2)\)
   (c) \(e^{2(t-1)}H(t - 1)\)
   (d) \(H(t - \pi/4)\sin(3(t - \pi/4))\)
   (e) \(t^2H(t - 1)\)
   (f) \(e^{-t}H(t - 2)\)

6. In this exercise, you will examine the effect of shifts in the time domain on the Laplace transform (graphically).
   (a) Sketch the graph of \(f(t) = \sin t\) in the time domain. Find the Laplace transform \(F(s) = \mathcal{L}\{f(t)\}(s)\). Sketch the graph of \(F\) in the \(s\)-domain on the interval \([0, 2]\).
   (b) Sketch the graph of \(g(t) = H(t - 1)\sin(t - 1)\) in the time domain. Find the Laplace transform \(G(s) = \mathcal{L}\{g(t)\}(s)\). Sketch the graph of \(G\) in the \(s\)-domain on the interval \([0, 2]\) on the same axes used to sketch the graph of \(F\).
   (c) Repeat the directions in part (b) for \(g(t) = H(t - 2)\sin(t - 2)\). Explain why engineers like to say that “a shift in the time domain leads to an attenuation (scaling) in the \(s\)-domain.”

7. Use the Heaviside function to concisely write each piecewise function.
   (a) \(f(t) = \begin{cases} 5 & 2 \leq t < 4; \\ 0 & \text{otherwise} \end{cases}\)
   (b) \(f(t) = \begin{cases} 0 & t < 0; \\ t & 0 \leq t < 3 \\ 4 & t \geq 3 \end{cases}\)
   (c) \(f(t) = \begin{cases} 0 & t < 0; \\ t^2 & 0 \leq t < 2 \\ 4 & t \geq 2 \end{cases}\)

8. Find the inverse Laplace transform of each function. Create a piecewise definition for your solution that doesn’t use the Heavyside function.
   (a) \(F(s) = \frac{e^{-2s}}{s + 3}\)
   (b) \(F(s) = \frac{1 - e^{-s}}{s^2}\)
   (c) \(F(s) = \frac{e^{-s}}{s^2 + 4}\)

9. For each initial value problem, sketch the forcing term, and then solve for \(y(t)\). Write your solution as a piecewise function (i.e., not using the Heavyside function). Recall that the function \(H_{ab}(t) = H(t - a) - H(t - b)\) is the interval function.
10. Define the function
\[ \delta_{\epsilon}^{p}(t) = \frac{1}{\epsilon}(H_{p}(t) - H_{p+\epsilon}(t)) \, . \]

(a) Show that the Laplace transform of \( \delta_{\epsilon}^{p}(t) \) is given by
\[ \mathcal{L}\{\delta_{\epsilon}^{p}(t)\} = e^{-sp} \frac{1 - e^{-s\epsilon}}{s\epsilon} \, . \]

(b) Use l’Hôpital’s rule to take the limit of the result in part (a) as \( \epsilon \to 0 \). How does this result agree with the fact that \( \mathcal{L}\{\delta_{p}(t)\} = e^{-sp} \)?

11. Use a Laplace transform to solve the following initial value problem:
\[ y' = \delta_{p}(t), \quad y(0) = 0 \]
How does your answer support what engineers like to say, that the “derivative of a unit step is a unit impulse”?

12. Define the function
\[ H_{\epsilon}^{p}(t) = \begin{cases} 
0, & 0 \leq t < p \\
\frac{1}{\epsilon}(t - p), & p \leq t < p + \epsilon \\
1, & t \geq p + \epsilon 
\end{cases} \]

(a) Sketch the graph of \( H_{\epsilon}^{p}(t) \).

(b) Without being too precise about things, we could argue that \( H_{\epsilon}^{p}(t) \to H_{p}(t) \) as \( \epsilon \to 0 \), where \( H_{p}(t) = H(t - p) \). Sketch the graph of the derivative of \( H_{\epsilon}^{p}(t) \).

(c) Compare your result in (b) with the graph of \( \delta_{\epsilon}^{p}(t) \). Argue that \( H_{\epsilon}^{p}(t) = \delta_{p}(t) \).

13. Solve the following initial value problems.
(a) \( y'' + 4y = H_{0}(t), \quad y(0) = 0, \quad y'(0) = 0 \)
(b) \( y'' + 4y' - 5y = H_{1}(t), \quad y(0) = 0, \quad y'(0) = 0 \)