1. Solve the following differential equations:
   (a) $y' - 2y = 0$
   (b) $y' - 2y = t - 3$
   (c) $y' - 2y = e^{3t}$
   (d) $y'' - 4y = 0$
   (e) $y'' + 4y = 0$
   (f) $y'' + 4y' + 3y = 10$.

2. The function
$$f(x) = \begin{cases} 0 & -\pi \le x < -\pi/2, \\ 1 & -\pi/2 \le x < \pi/2, \\ 0 & \pi/2 \le x \le \pi, \end{cases}$$
can be extended to be periodic of period 2π. Sketch the graph of the resulting function, and compute its Fourier series.

3. The function
$$f(t) = |x|, \qquad \text{for } x \in [-\pi, \pi]$$
can be extended to be periodic of period 2π. Sketch the graph of the resulting function, and compute its Fourier series.

4. The function
$$f(x) = \begin{cases} 0 & -\pi \le x < 0, \\ x & 0 \le x \le \pi, \end{cases}$$
can be extended to be periodic of period 2π. Sketch the graph of the resulting function, and compute its Fourier series.

5. Consider the 2π-periodic function defined by
$$f(x) = \begin{cases} x^2 & -\pi \le x < \pi, \\ f(x - 2k\pi), & -\pi + 2k\pi \le x < \pi + 2k\pi. \end{cases}$$
Sketch this function (at least for $k = -2, -1, 0, 1, 2$) and compute its Fourier series.

6. Find the Fourier series of the following functions *without* computing any integrals.
   (a) $f(x) = 2 - 3\sin 4x + 5\cos 6x$,
   (b) $f(x) = \sin^2 x$ [*Hint*: Use a standard trig identity.]

7. Determine which of the following functions are even, which are odd, and which are neither even nor odd:
   (a) $f(x) = x^3 + 3x$.
   (b) $f(x) = x^2 + |x|$.

(c) $f(x) = e^x$.

(d) $f(x) = \frac{1}{2}(e^x + e^{-x})$.

(e) $f(x) = \frac{1}{2}(e^x - e^{-x})$.

8. Suppose that f is a function defined on $\mathbb{R}$ (not necessarily periodic). Show that there is an odd function f_{odd} and an even function f_{even} such that $f(x) = f_{\text{odd}} + f_{\text{even}}$. [*Hint*: As a guiding example, suppose $f(x) = e^{ix}$, and consider $\cos x = \frac{1}{2}(e^{ix} + e^{-ix})$ and $i \sin x = \frac{1}{2}(e^{ix} - e^{-ix})$.]

9. Express the y-intercept of $f(x) = \dfrac{a_0}{2} + \displaystyle\sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$ in terms of the a_n's and b_n's. (*Hint*: It's *not* a_0 or $a_0/2$!)

10. Consider the 2π-periodic function $f(x) = \dfrac{a_0}{2} + \displaystyle\sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$. Write the Fourier series for the following functions:
    (a) The reflection of $f(x)$ across the y-axis;
    (b) The reflection of $f(x)$ across the x-axis;
    (c) The reflection of $f(x)$ across the origin.

11. (a) The Fourier series of an odd function consists only of sine-terms. What additional symmetry conditions on f will imply that the sine coefficients with even indices will be zero (i.e., each $b_{2n} = 0$)? Give an example of a non-zero function satisfying this additional condition.
    (b) What symmetry conditions on f will imply that the sine coefficients with odd indices will be zero (i.e., each $b_{2n+1} = 0$)? Give an example of a non-zero function satisfying this additional condition.
    (c) Sketch the graph of a non-zero even function, such that $a_{2n} = 0$ for all n.
    (d) Sketch the graph of a non-zero even function, such that $a_{2n+1} = 0$ for all n.

12. Consider the function defined on the interval $[0, \pi]$:

$$f(x) = \begin{cases} x & \text{for } 0 \le x < \pi/2, \\ \pi - x, & \text{for } \pi/2 \le x \le \pi. \end{cases}$$

    (a) Sketch the even extension of this function and find its Fourier cosine series.
    (b) Sketch the odd extension of this function and find its Fourier sine series.

13. Consider the function defined on the interval $[0, \pi]$:

$$f(x) = x(\pi - x).$$

    (a) Sketch the even extension of this function and find its Fourier cosine series.
    (b) Sketch the odd extension of this function and find its Fourier sine series.