## MthSc 208: Differential Equations (Summer II, 2010) In-class Worksheet 4c: Systems of differential equations (complex eigenvalues)

## NAME:

Consider the system of differential equations:  $\begin{cases} x_1' = -0.5x_1 + x_2, & x_1(0) = 0 \\ x_2' = -x_1 - 0.5x_2, & x_2(0) = 1 \end{cases}$ 

1. Write this in matrix form,  $\mathbf{x}' = \mathbf{A}\mathbf{x} + \mathbf{b}$ .

2. Given that the eigenvalues of **A** are  $\lambda_1 = -\frac{1}{2} + i$  and  $\lambda_2 = -\frac{1}{2} - i$ , with associated eigenvectors  $\mathbf{v}_1 = (1, i)$  and  $\mathbf{v}_2 = (1, -i)$ , write the general solution to  $\mathbf{x}' = \mathbf{A}\mathbf{x}$ .

3. Use Euler's formula  $(e^{it} = \cos t + i \sin t)$  to write a solution (e.g.,  $\mathbf{x}_1(t)$ ) as a sum of its real and imaginary parts:  $\mathbf{x}(t) = \mathbf{u}(t) + i\mathbf{w}(t)$ .

4. Write the general solution as a linear combination of real-valued functions:  $\mathbf{x}(t) = C_1 \mathbf{u}(t) + C_2 \mathbf{w}(t)$ .

5. Find the particular solution satisfying the initial condition.

6. Sketch the phase portrait of the system. Also sketch the particular solution satisfying the initial condition.

Written by M. Macauley 2