

**MthSc 208: Differential Equations (Summer II, 2010)**  
**In-class Worksheet 4c: Systems of differential equations (complex eigenvalues)**

NAME:

Consider the system of differential equations: 
$$\begin{cases} x_1' = -0.5x_1 + x_2, & x_1(0) = 0 \\ x_2' = -x_1 - 0.5x_2, & x_2(0) = 1 \end{cases}$$

1. Write this in matrix form,  $\mathbf{x}' = \mathbf{A}\mathbf{x} + \mathbf{b}$ .
2. Given that the eigenvalues of  $\mathbf{A}$  are  $\lambda_1 = -\frac{1}{2} + i$  and  $\lambda_2 = -\frac{1}{2} - i$ , with associated eigenvectors  $\mathbf{v}_1 = (1, i)$  and  $\mathbf{v}_2 = (1, -i)$ , write the general solution to  $\mathbf{x}' = \mathbf{A}\mathbf{x}$ .
3. Use Euler's formula ( $e^{it} = \cos t + i \sin t$ ) to write a solution (e.g.,  $\mathbf{x}_1(t)$ ) as a sum of its real and imaginary parts:  $\mathbf{x}(t) = \mathbf{u}(t) + i\mathbf{w}(t)$ .
4. Write the general solution as a linear combination of *real-valued* functions:  $\mathbf{x}(t) = C_1\mathbf{u}(t) + C_2\mathbf{w}(t)$ .

