MthSc 208: Differential Equations (Summer II, 2010)
In-class Worksheet 6c: Parseval's Identity

NAME:

Recall that Parseval's identity says that

$$\frac{1}{\pi}\int_{-\pi}^{\pi}\big(f(x)\big)^2\,dx = \frac{1}{2}a_0^2 + \sum_{n=1}^{\infty}(a_n^2 + b_n^2)\,.$$

We will use this to compute $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \cdots$.

1. Let $f(x) = x$ on $[-\pi, \pi]$ and extend $f(x)$ to be 2π-periodic. Write $f(x)$ as a Fourier series. (See Example 2 on pages 5-6 of the lecture notes.)

2. Compute $\displaystyle\frac{1}{\pi}\int_{-\pi}^{\pi}\big(f(x)\big)^2\,dx$. (The left-hand side of Parseval's identity.)

3. Compute $\displaystyle\frac{1}{2}a_0^2 + \sum_{n=1}^{\infty}(a_n^2 + b_n^2)$. (The right-hand side of Parseval's identity.)

4. Equate your answers to the previous two parts and solve for $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^2}$.