Recall that Parseval’s identity says that

\[ \frac{1}{\pi} \int_{-\pi}^{\pi} (f(x))^2 \, dx = \frac{1}{2} a_0^2 + \sum_{n=1}^{\infty} (a_n^2 + b_n^2). \]

We will use this to compute \( \sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \cdots. \)

1. Let \( f(x) = x \) on \( [-\pi, \pi] \) and extend \( f(x) \) to be \( 2\pi \)-periodic. Write \( f(x) \) as a Fourier series. (See Example 2 on pages 5-6 of the lecture notes.)

2. Compute \( \frac{1}{\pi} \int_{-\pi}^{\pi} (f(x))^2 \, dx. \) (The left-hand side of Parseval’s identity.)

3. Compute \( \frac{1}{2} a_0^2 + \sum_{n=1}^{\infty} (a_n^2 + b_n^2). \) (The right-hand side of Parseval’s identity.)

4. Equate your answers to the previous two parts and solve for \( \sum_{n=1}^{\infty} \frac{1}{n^2}. \)