1. (a) How many different 7-place license plates are possible if the first 2 places are letters and the other 5 are numbers?
   (b) What if no letter or number can be repeated in a license plate?

2. From 20 workers, 5 are selected and assigned to 5 different jobs. How many assignments are possible?

3. For years, telephone area codes in the United States and Canada consisted of a sequence of 3 digits. The first digit was a number between 2 and 9 (inclusive); the second digit was either a 0 or a 1; the third digit was a number between 1 and 9 (inclusive). How many area codes were possible? How many area codes starting with a 4 were possible?

4. Two experiments are to be performed. The first results in any one of m possible outcomes. If the first experiment results in outcome i, then the second experiment can result in any of n_i possible outcomes, for $i = 1, 2, \ldots, m$. How many possible outcomes are there for the two experiments?

5. The World Series in baseball coninues until either the American League team or the National league team wins four games. If the American League team wins, how many different orders (of wins and losses) are possible if the series goes:
   (a) four games
   (b) five games
   (c) six games
   (d) seven games
   (e) How many total possible outcomes for a world series?

6. Consider a standard 52 card poker deck. Find the number of 5 card hands with
   (a) All 5 cards the same suit (there are 4 suits; each suit has 13 cards)
   (b) Two pairs (e.g. 2 Aces and 2 Kings and a third card which is not an Ace nor a King)
   (c) Four of a kind (e.g. all four 2's)

7. A dance class has 22 students, 10 women and 12 men. If 5 men and 5 women are to be selected and then paired off, how many results are possible?

8. From a group of 8 women and 6 men a committee consisting of 3 men and 3 women is to be formed. How many different committees are possible if
   (a) 2 of the men refuse to serve together
   (b) 2 of the women refuse to serve together
   (c) 1 man and 1 woman (ex's) refuse to serve together

9. A small probability class has 7 students. The professor has decided to give 1 'A', 4 'B's, and 2 'C's. How many ways can grades be assigned?

10. You're throwing a party and want to buy ten 2 liter bottles of soda. The local store has Coke, Sprite, Dr. Pepper, and Mountain Dew. How many ways can you do this?

11. (a) Write out the first 8 lines of Pascal's triangle.

    (b) Repeat part (a), but write the entires as binomial coefficients (e.g. $\binom{n}{r}$, instead of numbers).

    (c) The following identity should now be clear:

$$\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r} .$$

    Give a *combinatorial proof* of this fact, by counting – two different ways – the number of size-r groups that can be formed from n people, one of whom is a "weirdo."

12. (a) Give a combinatorial proof that

$$\binom{n+m}{r} = \binom{n}{0}\binom{m}{r} + \binom{n}{1}\binom{m}{r-1} + \cdots + \binom{n}{r}\binom{m}{0} .$$

    Hint: Consider a group consisting of n men and m women.

    (b) Use part (a) to prove that

$$\binom{2n}{n} = \sum_{k=0}^{n} \binom{n}{k}^2 .$$

13. (a) Use the binomial theorem to show that

$$\sum_{k=0}^{n} (-1)^k \binom{n}{k} = 0 .$$

    (b) Give an interpretation of this fact in terms of the entries of Pascal's triangle. Viewing it this way, does it surprise you? (Look at both the even and odd rows.)

14. Use the binomial theorem to expand $(3x^2 + y)^5$.