1. Let X be a random variable with probability desnsity function

$$f(x) = \begin{cases} c(1-x^2) & -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

   (a) What is the value of c?

   (b) What is the cumulative distribution function of X?

2. Consider the function

$$f(x) = \begin{cases} C(2x-x^3) & 0 < x < \frac{5}{2} \\ 0 & \text{otherwise} \end{cases}$$

   Could f be a probability density function? If so, determine C. Repeat if $f(x)$ were given by

$$f(x) = \begin{cases} C(2x-x^2) & 0 < x < \frac{5}{2} \\ 0 & \text{otherwise} \end{cases}$$

3. A filling station is supplied with gasoline once a week. If its weekly volume of sales in thousands of gallons is a random variable with probability density function

$$f(x) = \begin{cases} 5(1-x)^4 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

   What must the capacity of the tank be so that the probability of the supply's being exhausted in a given week is 0.01?

4. Compute $E[X]$ if X has a density function given by

   (a) $f(x) = \begin{cases} \frac{1}{4}xe^{-x/2} & x > 0 \\ 0 & \text{otherwise} \end{cases}$

   (b) $f(x) = \begin{cases} c(1-x^2) & -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$

   (c) $f(x) = \begin{cases} 5x^{-2} & x > 5 \\ 0 & x \leq 5 \end{cases}$

5. You arrive at a bus stop at 10:00, knowing that the bus will arrive at some time uniformly distributed between 10 and 10:30.

   (a) What is the probability that you will have to wait longer than 10 minutes?

   (b) If, at 10:15, the bus has not yet arrived, what is the probability that you will have to wait at least an additional 10 minutes?

6. If X is a normal random variable with parameters $\mu = 10$ and $\sigma^2 = 36$, compute

   (a) $P\{X > 5\}$;

   (b) $P\{4 < X < 16\}$;

   (c) $P\{X < 8\}$;

(d) $P\{X < 20\}$;

(e) $P\{X > 16\}$.

7. Suppose that X is a normal random variable with mean 5. If $P\{X > 9\} = 0.2$, approximately what is $\text{Var}(X)$?

8. Let X be a normal random varaible with mean 12 and variance 4. Find the value of c such that $P\{X > c\} = 0.1$.

9. If 65% of the population of a large community is in favor of a proposed rise in school taxes, approximate the probability that a random sample of 100 people will contain

   (a) at least 50 who are in favor of the proposition;

   (b) between 60 and 70 inclusive who are in favor;

   (c) fewer than 75 in favor.

10. One thousand independent rolls of a fair die will be made. Compute an approximation to the probability that the number 6 will appear between 150 and 200 times inclusively. If the number 6 appears exactly 200 times, find the probability that the number 5 will appear less than 150 times?

11. In 10,000 independent tosses of a coin, the coin landed on heads 5800 times. Is it reasonable to assume that the coin is not fair? Explain your reasoning.

12. Recall that the standard deviation of X, denoted $\text{SD}(X)$, is given by $\text{SD}(X) = \sqrt{\text{Var}(X)}$. Find $\text{SD}(aX + b)$ if X has variance σ^2.