1. Let $f \in \mathcal{B}[a,b]$, and suppose that $\mathcal{P}$ and $\mathcal{P}^*$ are partitions of $[a,b]$ with $\mathcal{P}^* \supset \mathcal{P}$. Prove that $\mathcal{U}(\mathcal{P}^*, f) \leq \mathcal{U}(\mathcal{P}, f)$.

2. The method used in Example 6.1.6 (d) can be summarized as the following theorem:

   Let $f \in \mathcal{B}[a,b]$ and suppose there exists a sequence $\{\mathcal{P}_n\}_{n=1}^{\infty}$ of partitions of $[a,b]$ such that
   $$\lim_{n\to\infty} \mathcal{L}(\mathcal{P}_n, f) = \lim_{n\to\infty} \mathcal{U}(\mathcal{P}_n, f) = L \in \mathbb{R}.$$
   Then $f \in \mathcal{R}[a,b]$ and $\int_a^b f = L$. Prove this.

3. Use the above theorem to compute $\int_a^b x\, dx$.

4. For each of the given functions, check the Riemann integrability and evaluate the Riemann integral on the specificed interval.

   (a) $f(x) = \begin{cases} -1, & x \in [0,1) \\ 2, & x \in [1,2] \end{cases}$ on $[0,2]$. (b) $f(x) = 2x+1$ on $[0,1]$.

5. Let $f(x) = \begin{cases} 0, & x \in \mathbb{Q} \cap [0,1] \\ x, & x \in \mathbb{Q}^c \cap [0,1]. \end{cases}$ Compute $\underline{\int_0^1} f$ and $\overline{\int_0^1} f$ and determine if $f \in \mathcal{R}[0,1]$.