1. Let $P(t)$ be the net worth of an investment after $t$ years, that is growing at a 5% rate. Suppose that after two years, the investment is worth \$200. Write down an *initial value problem* (the differential equation & initial condition) for $P$, and sketch its solution.

2. Let $T(t)$ be the temperature of a cup of water $t$ minutes after being placed in a room where the ambient temperature is $72°$.

   (a) Write down a differential equation that $T$ satisfies.

   (b) Sketch the solution curve of the solution satisfying $T(0) = 100$.

   (c) Sketch the solution curve of the solution satisfying $T(0) = 40$.

   (d) Sketch the solution curve of the solution satisfying $T(0) = 72$.

3. Repeat the previous exercise, except let $T(t)$ be the temperature of a sheet of metal (which cools down and heats up *much* quicker than water). Qualitatively, what is the difference between the solution curves in these two problems? Which value of $k$ is bigger and why?

4. Sketch the slope field of the ODE $y' = t - 2y$ using the isocline method for $y' = c$, for $c = 0, \pm 1, \pm 2, \pm 3$. Sketch the particular solutions that satisfy $y(0) = 1$ and $y(2) = 2$.

5. Sketch the slope field of the ODE $y\,y' = -t$ using the isocline method for $c = 0, \pm\frac{1}{2}, \pm 1, \pm 2$, and sketch the particular solution that satisfies $y(0) = 1$.

6. Let $y' = f(y, t)$ be an ODE. Explain why two solution curves in its slope field can never cross.

7. Sketch the steady-state (constant) solutions of $y' = 6 + y - y^2$ in the $ty$-plane. These solutions divide the plane into regions. Sketch at least one solution curve in each of these region.

8. Consider the differential equation $y' = y(4 - y)$.

   (a) Show that $y(t) = 4/(1+Ce^{-4t})$ is a solution for any value of $C$ by plugging it into the ODE. This family of solutions is called a *general solution* to the differential equation.

   (b) Sketch the solutions for $C = 1, 2, \ldots, 5$. (Hint: This ODE is autonomous).

   (c) What are the steady-state (constant) solutions?

   (d) The general solution may fail to produce all solutions of a differential equation. Find a solution that is not given by any value of $C$. (Hint: Look at part (c)).

   (e) Describe a physical situation that this differential equation could model, and justify your reasons. (Hint: Consider population growth).

9. Consider the initial value problem $y' = t + y$, $y(0) = 1$.

   (a) When computing a solution by hand using Euler's method, it is beneficial to arrange your work in a table, as shown below where the first step is computed. Continue with Euler's method using step-size $h = 0.1$ and complete all missing entries of the table.

| $k$ | $t_k$ | $y_k$ | $f(t_k, y_k) = t_k + y_k$ | $h$ | $f(t_k, y_k) \cdot h$ |
|---|---|---|---|---|---|
| 0 | 0.0 | 1.0 | 1.0 | 0.1 | 0.1 |
| 1 | 0.1 | 1.1 | | | |
| 2 | 0.2 | | | | |
| 3 | 0.3 | | | | |
| 4 | 0.4 | | | | |
| 5 | 0.5 | | | | |

(b) The general solution of $y' = t + y$ is $y(t) = Ce^t - t - 1$. Using this, compute the actual value of $y(0.5)$.

10. Consider the initial value problem $y' = (1 + t)y$, $y(0) = -1$.

(a) Use Euler's method to approximate $y(1)$, for step-size $h = 0.2$, and then for $h = 0.1$. Arrange your results in the tabular form as in the previous exercise.

(b) Compute the actual value of $y(1)$ by solving the initial value problem $y' = (1 + t)y$, $y(0) = -1$ and plugging in $t = 1$.