

1. Solve for t , and simplify whenever possible.

(a) $3e^{-4t} = 5$

(b) $2 = e^3 \cdot e^{2t}$

(c) $t^2 = e^6$

(d) $(\frac{4}{3})^{-t} = 7$

(e) $e^{\frac{1}{3} \ln t} = 27$

(f) $e^{-\frac{1}{3} \ln t} = 27$

2. Compute the following integrals:

(a) $\int \frac{1}{2t} dt$

(b) $\int \frac{1}{3-4t} dt$

3. Find the general solution of the following differential equations.

(a) $y' = ty$

(b) $ty' = -2y$

(c) $y' = e^{t-y}$

4. Suppose that \$1200 is invested at a rate of 5%, compounded continuously.

(a) Assuming no additional withdrawals or deposits, how much will be in the account after 10 years?

(b) How long will it take the balance to reach \$5000?

5. Tritium is an isotope of hydrogen that is sometimes used as a biochemical tracer. Suppose that 100 mg of tritium decays to 80 mg in 4 hours. Determine its half-life.

6. Suppose a cold beer at 40°F is placed into a warm room at 70°F. Suppose 10 minutes later, the temperature of the beer is 48°F. Use Newton's law of cooling to find the temperature 25 minutes after the beer was placed into the room.

7. A murder victim is discovered at midnight at the temperature of the body is recorded at 31°C. One hour later, the temperature of the body is 29°C. Assume that the surrounding air temperature remains constant at 21°C. Use Newton's law of cooling (the differential equation $T' = k(A - T)$) to calculate the victim's time of death (when his body temperature was 37°C).

8. A parachutist of mass 60 kg free-falls from an airplane at an altitude of 5000 meters. He is subjected to an air resistance force proportional to his speed. Assume that the constant of proportionality is $r = 10$ kg/sec.

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- (a) Find and solve the differential equation governing the velocity of the parachuter at time t seconds after the start of his free-fall.
- (b) Assuming he does not deploy his parachute, find his limiting velocity and how much time will elapse before he hits the ground (you may need to use a computer for this last part, a visual approximation from the appropriate graph is fine).
9. In our model of air resistance, the resistance force has depended only on the velocity. However, for an object that drops a considerable distance, such as the parachutist in the previous exercise, there is a dependence on the altitude as well. It is reasonable to assume that the resistance force is proportional to air pressure, as well as to the velocity. Furthermore, to a first-order approximation, the air pressure varies exponentially with the altitude (i.e., it is proportional to e^{-ax} , where a is a constant and x is the altitude). Propose and justify (*but do not solve!*) a differential equation model for the velocity of a falling object subject to such a resistance force.