1. For each of the first-order differential equations, decide whether it is linear or nonlinear. If the equation is linear, state whether it is homogeneous or inhomogeneous.

   (a) $y' = ky$

   (b) $y' = k(72 - y)$

   (c) $y' = y(4 - y)$

   (d) $y' = t + y$

   (e) $3y' + 5y = 3\cos 2t$

   (f) $3y' + 5y = 3\cos 2y$

   (g) $y' = 4t^2 y - \sin t$

   (h) $y' = 4ty^2 - \sin t$

2. Use the integrating factor method to find the general solution of the following differential equations.

   (a) $2y' - 3y = 5$

   (b) $y' + 2ty = 5t$

   (c) $ty' = 4y + t^4$

3. Use the varation of parameters method to find the general solution of the following differential equation. Then find the particular solution satisfying the given initial condition.

   (a) $y' - 3y = 4, \quad y(0) = 2$

   (b) $y' + y = e^t, \quad y(0) = 1$

   (c) $y' + 2ty = 2t^3, \quad y(0) = -1.$

4. A murder victim is discovered at midnight at the temperature of the body is recorded at 31°C, and it was discovered that the proportionality constant in Newton's law was $k = \ln(5/4)$. Assume that at midnight the surrounding air temperature $A(t)$ is 0°C, and is falling at a constant rate of 1°C per hour. At what time did the victim die? (Set $T(t) = 37$ and solve for t – use a computer or calculator for this part.) [*Hint*: Letting $t = 0$ represent midnight will simplify your calculations.]

5. Consider Newton's law of cooling, but suppose that the ambient temperature varies sinusoidally with time, as in
$$T' = k(A\sin\omega t - T)\,.$$

   (a) Solve the homogeneous equation, $T_h' = -kT_h$.

   (b) The ODE above is not autonomous, so finding a particular solution T_p is a bit more difficult (there is no steady-state solution). However, it doesn't hurt to guess. As a first guess, substitute $T_p = C\cos\omega t + D\sin\omega t$ into the equation $T' + kT = kA\sin\omega t$ and equate coefficients of the sine and cosine terms, and show that
$$-\omega C + kD = kA \qquad \text{and} \qquad kC + \omega D = 0\,.$$

(c) Solve the simultaneous equations in part (b), and determine the general solution to this ODE.

(d) Give a qualitative physical description of what the particular solution T_p represents, and why. [Hint: Consider the long-term behavior of the temperature $T(t)$.]

6. Suppose that the temperature T inside a mountain cabin behaves according to Newton's law of cooling, as in

$$T' = \frac{1}{2}(A(t) - T)\,,$$

where t is measured in hours and the ambient temperature $A(t)$ outside the cabin varies sinusoidally with a period of 24 hours. At 6am, the ambient temperature outside is at a minimum of 40°, and at 6pm, the ambient temperature is at a maximum of 80°.

(a) Adjust the differential equation above to model the sinusoidal nature of the ambient temperature.

(b) Suppose that at noon the temperature inside the cabin is 50°. Solve the resulting initial value problem. [Hint: Use the formula you derived in part (c) of the previous problem! Letting $t = 0$ represent noon will simplify your calculations.]

(c) Use a computer to sketch the graph of the temperature inside the cabin. On the same coordinate system, superimpose the plot of the ambient temperature outside the cabin. Comment on the appearance of the plot.

7. Solve the differential equation $y' = 2y + 4$ four different ways:

(a) Undetermined coefficients (that is, finding a constant solution, and writing $y(t) = y_h(t) + y_p(t)$).

(b) Integrating factor

(c) Variation of parameters

(d) Separation of variables

8. Solve the following ODEs using the method of undetermined coefficients: That is, write $y(t) = y_h(t)+y_p(t)$, then solve the homogeneous equation, then guess a particular solution, and add those together to get the general solution.

(a) $y' - 2y = 0$

(b) $y' - 2y = 10$

(c) $y' - 2y = t$

(d) $y' - 2y = t^2 + 1$

(e) $y' - 2y = 4e^{3t}$

(f) $y' - 2y = 5 \sin 3t$

9. A tank contains 100 gal of pure water. A salt solution with concentration 3 lb/gal enters the tank at a rate of 2 gal/min. Solution drains from the tank at a rate of 2 gal/min. Sketch a picture of this situation, then *without doing any math*, determine the eventual concentration of the salt solution in the tank (i.e., the steady-state solution).

10. A tank contains 100 gal of pure water. At time zero, a sugar-water solution containing 0.2 lb of sugar per gal enters the tank at a rate of 3 gal per minute. Simultaneously, a drain is opened at the bottom of the tank allowing the sugar solution to leave the tank at 3 gal per minute. Assume that the solution in the tank is kept perfectly mixed at all time.

    (a) What will be the sugar content in the tank after 20 minutes?

    (b) How long will it take the sugar content in the tank to reach 15 lb?

    (c) What will be the eventual sugar content in the tank?

11. A tank contains 500 gal of a salt-water solution containing 0.05 lb of salt per gallon of water. Pure water is poured into the tank and a drain at the bottom of the tank is adjusted so as to keep the volume of solution in the tank constant. At what rate (gal/min) should the water be poured into the tank to lower the salt concentration to 0.01 lb/gal of water in under one hour (i.e., $t = 60$ minutes)?

12. A tank initially contains 100 gal of a salt-water solution containing 0.05 lb of salt for each gallon of water. At time zero, pure water is poured into the tank at a rate of 2 gal per minute. Simultaneously, a drain is opened at the bottom of the tank that allows the salt-water solution to leave at a rate of 3 gal per minute. What will be the salt content in the tank when precisely 50 gal of salt solution remain.