1. Find the Laplace transform of the following functions by using a table of Laplace transforms

   (a) $f(t) = -2$

   (b) $f(t) = e^{-2t}$

   (c) $f(t) = \sin 3t$

   (d) $f(t) = te^{-3t}$

   (e) $f(t) = e^{2t}\cos 2t$

2. Transform the given initial value problem into an algebraic equation involving $Y(s) := \mathcal{L}(y)$, and solve for $Y(s)$.

   (a) $y'' + y = \sin 4t, \quad y(0) = 0, \quad y'(0) = 1$

   (b) $y'' + y' + 2y = \cos 2t + \sin 3t, \quad y(0) = -1, \quad y'(0) = 1$

   (c) $y' + y = e^{-t}\sin 3t, \quad y(0) = 0$

3. Find the inverse Laplace transform of the following functions.

   (a) $Y(s) = \dfrac{2}{3-5s}$

   (b) $Y(s) = \dfrac{1}{s^2+4}$

   (c) $Y(s) = \dfrac{5s}{s^2+9}$

   (d) $Y(s) = \dfrac{3}{s^2}$

   (e) $Y(s) = \dfrac{3s+2}{s^2+25}$

   (f) $Y(s) = \dfrac{2-5s}{s^2+9}$

   (g) $Y(s) = \dfrac{s}{(s+2)^2+4}$

   (h) $Y(s) = \dfrac{3s+2}{s^2+4s+29}$

   (i) $Y(s) = \dfrac{2s-2}{(s-4)(s+2)}$

   (j) $Y(s) = \dfrac{3s^2+s+1}{(s-2)(s^2+1)}$

4. Use the Laplace transform to solve the following initial value problems.

   (a) $y' - 4y = e^{-2t}t^2, \quad y(0) = 1$

   (b) $y'' - 9y = -2e^t, \quad y(0) = 0, \quad y'(0) = 1$

5. Find the Laplace transform of the given functions.
   (a) $3H(t-2)$
   (b) $(t-2)H(t-2)$
   (c) $e^{2(t-1)}H(t-1)$
   (d) $H(t-\pi/4)\sin 3(t-\pi/4)$
   (e) $t^2H(t-1)$
   (f) $e^{-t}H(t-2)$

6. In this exercise, you will examine the effect of shifts in the time domain on the Laplace transform (graphically).
   (a) Sketch the graph of $f(t) = \sin t$ in the time domain. Find the Laplace transform $F(s) = \mathcal{L}\{f(t)\}(s)$. Sketch the graph of F in the s-domain on the interval $[0, 2]$.
   (b) Sketch the graph of $g(t) = H(t-1)\sin(t-1)$ in the time domain. Find the Laplace transform $G(s) = \mathcal{L}\{g(t)\}(s)$. Sketch the graph of G in the s-domain on the interval $[0, 2]$ on the same axes used to sketch the graph of F.
   (c) Repeat the directions in part (b) for $g(t) = H(t-2)\sin(t-2)$. Explain why engineers like to say that "a shift in the time domain leads to an attenuation (scaling) in the s-domain."

7. Use the Heaviside function to concisely write each piecewise function.
   (a) $f(t) = \begin{cases} 5 & 2 \leq t < 4; \\ 0 & \text{otherwise} \end{cases}$
   (b) $f(t) = \begin{cases} 0 & t < 0; \\ t & 0 \leq t < 3 \\ 4 & t \geq 3 \end{cases}$
   (c) $f(t) = \begin{cases} 0 & t < 0; \\ t^2 & 0 \leq t < 2 \\ 4 & t \geq 2 \end{cases}$

8. Find the inverse Laplace transform of each function. Create a piecewise definition for your solution that doesn't use the Heavyside function.
   (a) $F(s) = \dfrac{e^{-2s}}{s+3}$
   (b) $F(s) = \dfrac{1-e^{-s}}{s^2}$
   (c) $F(s) = \dfrac{e^{-s}}{s^2+4}$

9. For each initial value problem, sketch the forcing term, and then solve for $y(t)$. Write your solution as a piecewise function (i.e., not using the Heavysie function). Recall that the function $H_{ab}(t) = H(t-a) - H(t-b)$ is the interval function.

(a) $y'' + 4y = H_{01}(t), \quad y(0) = 0, \;\; y'(0) = 0$

(b) $y'' + 4y = t\,H_{01}(t), \quad y(0) = 0, \;\; y'(0) = 0$

10. Define the function

$$\delta_p^\epsilon(t) = \frac{1}{\epsilon}\left(H_p(t) - H_{p+\epsilon}(t)\right).$$

(a) Show that the Laplace transform of $\delta_p^\epsilon(t)$ is given by

$$\mathcal{L}\left\{\delta_p^\epsilon(t)\right\} = e^{-sp}\,\frac{1 - e^{-s\epsilon}}{s\epsilon}.$$

(b) Use l'Hôpital's rule to take the limit of the result in part (a) as $\epsilon \to 0$. How does this result agree with the fact that $\mathcal{L}\{\delta_p(t)\} = e^{-sp}$?

11. Use a Laplace transform to solve the follosing initial value problem:

$$y' = \delta_p(t), \qquad y(0) = 0$$

How does your answer support what engineers like to say, that the "derivative of a unit step is a unit impulse"?

12. Define the function

$$H_p^\epsilon(t) = \begin{cases} 0, & 0 \le t < p \\ \frac{1}{\epsilon}(t-p), & p \le t < p+\epsilon \\ 1, & t \ge p+\epsilon \end{cases}$$

(a) Sketch the graph of $H_p^\epsilon(t)$.

(b) Without being too precise about things, we could argue that $H_p^\epsilon(t) \to H_p(t)$ as $\epsilon \to 0$, where $H_p(t) = H(t-p)$. Sketch the graph of the derivative of $H_p^\epsilon(t)$.

(c) Compare your result in (b) with the graph of $\delta_p^\epsilon(t)$. Argue that $H_p'(t) = \delta_p(t)$.

13. Solve the following initial value problems.

(a) $y'' + 4y = \delta(t), \quad y(0) = 0, \;\; y'(0) = 0$

(b) $y'' - 4y' - 5y = \delta(t), \quad y(0) = 0, \;\; y'(0) = 0$