

MthSc 208: Differential Equations (Summer II, 2012)
In-class Worksheet 4d: Systems of differential equations (repeated eigenvalues)

NAME:

Consider the system of differential equations: $\begin{cases} x_1' = -x_1 - x_2 \\ x_2' = x_1 - 3x_2 \end{cases}$

1. Write this in matrix form, $\mathbf{x}' = \mathbf{A}\mathbf{x} + \mathbf{b}$.
2. Knowing that \mathbf{A} has a repeated eigenvalue, $\lambda_{1,2} = -2$, and one eigenvector, $\mathbf{v}_1 = (1, 1)$, write down a solution $\mathbf{x}_1(t)$ to $\mathbf{x}' = \mathbf{A}\mathbf{x}$.
3. To find a second solution, assume that $\mathbf{x}_2(t) = te^{\lambda t}\mathbf{v} + e^{\lambda t}\mathbf{w}$. Plug this back into $\mathbf{x}' = \mathbf{A}\mathbf{x}$ and equate coefficients (of $te^{-\lambda t}$ and $e^{\lambda t}$) to get a system of two equations, involving \mathbf{v} , \mathbf{w} , and \mathbf{A} .

