Read: Francis Su's essay on good mathematical writing (posted on the course website).

1. Answer the following questions from the directly from the essay on good mathematical writing.

   (a) What is a good rule of thumb for what you should assume of your audience as you write your homework sets?

   (b) Is chalkboard writing formal or informal writing?

   (c) Why is the proof by contradiction on page 3 not really a proof by contradiction?

   (d) Name three things a lazy writer would do that a good writer would not.

   (e) What's the difference in meaning between these three phrases?

   "Let $A = 12$."   "So $A = 12$."   "$A = 12$."

2. Let $X = \mathbb{Z} \setminus \mathbb{Z}_{\neq 0} = \{(m, n) \mid m, n \in \mathbb{Z},\ n \neq 0\}$. That is, X consists of all *ordered pairs* (m, n) of integers with $n \neq 0$. Consider an *equivalence relation* $\sim$ on X defined as

$$a/b \sim c/d \quad \text{if} \quad ad = bc.$$

   The set $\mathbb{Q} := X/\sim$ of equivalence classes are called the *rational numbers*. We typically denote the class $[m/n]$ by any representative, e.g., m/n.

   (a) Define *addition* of rational numbers by

$$a/b + c/d = [ad + bc]/[bd].$$

   Show that this is *well-defined*, which means that it does *not* depend on the choice of representative. Specifically, show that if $w/x \sim a/b$ and $y/z \sim c/d$, then $w/x + y/z = a/b + c/d$.

   (b) Define a *multiplication* of rational numbers, and show that it is well-defined.

3. Prove that there is no rational number whose square is 12.

4. *Moved to HW 2, but useful for proving the previous problem – this is Exercise 1.1 in Rudin.* If r is rational ($r \neq 0$) and x is irrational, prove that $r + x$ and rx are irrational. Do not make any references to Dedekind cuts – assume that x being irrational means only that it cannot be written as a fraction of two integers.