Read: Rudin, Chapter 1. Do not wory about understanding every detail, but get a good sense of the big ideas.

1. If r is rational ($r \neq 0$) and x is irrational, prove that $r + x$ and rx are irrational. Do not make any references to Dedekind cuts – assume that x being irrational means only that it cannot be written as a fraction of two integers.

2. Prove Proposition 1.15 in Rudin: The axioms for multiplication (see p. 5) imply the following statements.
   (a) If $x \neq 0$ and $xy = xz$ then $y = z$.
   (b) If $x \neq 0$ and $xy = x$ then $y = 1$.
   (c) If $x \neq 0$ and $xy = 1$ then $y = 1/x$.
   (d) If $x \neq 0$ then $1/(1/x) = x$.

3. Let E be a nonempty subset of an ordered set. Suppose α is a lower bound of E and β is an upper bound of E. Prove that $\alpha \leq \beta$.

4. Let A be a nonempty set of real numbers which is bounded below. Let $-A$ be the set of all numbers $-x$, where $x \in A$. Prove that $\inf A = -\sup(-A)$.

5. For any real number $a \in \mathbb{R}$ and nonempty $B \subset \mathbb{R}$, define the set $a + B = \{a + b : b \in B\}$. Show that if B is bounded above, then $\sup(a + B) = a + \sup B$.

6. Let u be an upper bound of a non-empty set A in $\mathbb{R}$. Prove that u is the supremum of A if and only if for all $\epsilon > 0$, there is an $a \in A$ such that $u - \epsilon < a$. Formulate (but do not prove) an analogous statement for the infimum of A.

7. Let A, B be subsets of $\mathbb{R}$ that are bounded above, and let $A+B = \{a+b : a \in A,\ b \in B\}$. Show that
$$\sup(A + B) = \sup A + \sup B.$$