Read: Rudin, Chapter 2, pages 24–30.

1. Prove that the Principle of Induction implies the Well Ordering Princple for $\mathbb{N}$. *Hint*: Let $L(n)$ be the statement "if $A \subset \mathbb{N}$ that contains a number $m \leq n$, then A has a least element," and induct on n.

2. A complex number n is said to be *algebraic* if there are integers $a_0, \dots, a_n$, not all zero, such that
$$a_n z^n + a_1 z^{n-1} + \cdots + a_1 z + a_0 = 0\,.$$
   (a) Prove that the set $\mathbb{A}$ of algebraic numbers is countable. *Hint*: For every positive integer N, there are only finitely many equations with
$$n + |a_0| + |a_1| + \cdots + |a_n| = N\,.$$
   (b) Prove that there exist real numbers which are not algebraic.

3. Is the set of all irrational real numbers countable? Prove or disprove.

4. For $x, y \in \mathbb{R}$, define
$$d_1(x,y) = (x-y)^2, \quad d_2(x,y) = |x^2 - y^2|, \quad d_3(x,y) = |x - 2y|, \quad d_4(x,y) = \frac{|x-y|}{1+|x-y|}.$$
Determine, for each of these, whether it is a metric or not. That is, either verify that the three required conditions hold, or show by example that one of them fails.

5. Construct a bounded set of real numbers with exactly three limit points.