Read: Rudin, Chapter 2, pages 36–40.

1. Let $A_1, A_2, A_3, \ldots$ be subsets of a metric space.
   (a) If $B_n = \bigcup_{i=1}^{n} A_i$, prove that $\overline{B_n} = \bigcup_{i=1}^{n} \overline{A_i}$, for $n = 1, 2, 3, \ldots$.
   (b) If $B = \bigcup_{i=1}^{\infty} A_i$, prove that $\overline{B} \supset \bigcup_{i=1}^{\infty} \overline{A_i}$.

2. Is every point of every open set $E \subset \mathbb{R}^2$ a limit point of $E$? Answer the same question for closed sets in $\mathbb{R}^2$.

3. Let $E^\circ$ denote the set of all interior points of a set $E$, which we call the interior of $E$.
   (a) Prove that $E^\circ$ is always open.
   (b) Prove that $E$ is open if and only if $E^\circ = E$.
   (c) If $G \subset E$ and $G$ is open, prove that $G \subset E^\circ$.
   (d) Prove that the complement of $E^\circ$ is the closure of the complement of $E$.
   (e) Do $E$ and $\overline{E}$ always have the same interiors? Prove or disprove.
   (f) Do $E$ and $E^\circ$ always have the same closures? Prove or disprove.

4. Let $X$ be an infinite set. For $p, q \in X$, define
   \[ d(p, q) = \begin{cases} 
   1 & \text{if } p \neq q \\
   0 & \text{if } p = q.
   \end{cases} \]
   Prove that this is a metric. Which subsets of the resulting metric space are open? Which are closed? Which are compact? Prove all your claims.

5. Let $K = \{1/n : n \in \mathbb{N}\} \cup \{0\}$. Prove that $K$ is compact.