Read: Rudin, Chapter 2, pages 36–40.

1. Let A_1, A_2, A_3,... be subsets of a metric space.
   (a) If $B_n = \bigcup_{i=1}^{n} A_i$, prove that $\bar{B}_n = \bigcup_{i=1}^{n} \bar{A}_i$, for $n = 1, 2, 3, \ldots$.
   (b) If $B = \bigcup_{i=1}^{\infty} A_i$, prove that $\bar{B} \supset \bigcup_{i=1}^{\infty} \bar{A}_i$.

2. Is every point of every open set $E \subset \mathbb{R}^2$ a limit point of E? Answer the same question for closed sets in $\mathbb{R}^2$.

3. Let E° denote the set of all interior points of a set E, which we call the *interior* of E.
   (a) Prove that E° is always open.
   (b) Prove that E is open if and only if $E^\circ = E$.
   (c) If $G \subset E$ and G is open, prove that $G \subset E^\circ$.
   (d) Prove that the complement of E° is the closure of the complement of E.
   (e) Do E and $\bar{E}$ always have the same interiors? Prove or disprove.
   (f) Do E and E° always have the same closures? Prove or disprove.

4. Let X be an infinite set. For $p, q \in X$, define

$$d(p,q) = \begin{cases} 1 & \text{if } p \neq q \\ 0 & \text{if } p = q. \end{cases}$$

   Prove that this is a metric. Which subsets of the resulting metric space are open? Which are closed? Which are compact? Prove all your claims.

5. Let $K = \{1/n : n \in \mathbb{N}\} \cup \{0\}$. Prove that K is compact.