Read: Rudin, Chapter 4.

1. Let f and g be continuous mappings of a metric space X into a metric space Y, and let E be a dense subset of X.
   (a) Prove that $f(E)$ is dense in $f(X)$.
   (b) If $g(p) = f(p)$ for all $p \in E$, prove that $g(p) = f(p)$ for all $p \in X$. (In other words, a continuous mapping is determined by its values on a dense subset of its domain.)

2. Let $f\colon X \to Y$ and $g\colon Y \to Z$ be uniformly continuous between metric spaces.
   (a) Prove that if $\{x_n\}$ is a Cauchy sequence in X, then $\{f(x_n)\}$ is Cauchy in Y.
   (b) Prove that $g \circ f$ is uniformly continuous.

3. Prove the following fixed point theorem: If $f\colon [0,1] \to [0,1]$ is continuous, then $f(x) = x$ for at least one $x \in [0,1]$. [*Hint*: Consider the function $g(x) := f(x) - x$.]

4. Every $x \in \mathbb{Q}$ can be written as $x = m/n$, where $n > 0$, and m and n are integers with no common divisors. When $x = 0$, we take $n = 1$. Consider the real-valued function f defined by
$$f(x) = \begin{cases} 0 & x \notin \mathbb{Q}, \\ 1/n & x = m/n. \end{cases}$$
   Prove that f is continuous at every irrational point, and that f has a *simple discontinuity* at every rational point.