1. In the problems below, the coefficient matrix contains a parameter $\alpha$. For both of these matrices, carry out the following steps.

   (i) Determine the eigenvalues in terms of $\alpha$.
   (ii) Find the critical value or values of $\alpha$ where the qualitative nature of the phase portrait for the system changes.
   (iii) Draw a phase portrait for a value of $\alpha$ slight below, and for another value slightly above, each critical value.
   (iv) Draw a phase portrait when $\alpha$ is exactly the critical value.

   (a) $\begin{bmatrix} \alpha & 1 \\ -1 & \alpha \end{bmatrix} \mathbf{x}$
   (b) $\begin{bmatrix} -1 & \alpha \\ -1 & -1 \end{bmatrix} \mathbf{x}$

2. Find the general solution for each of the given systems and draw a phase portrait. Describe the behavior of the solutions as $t \to \infty$ and as $t \to -\infty$.

   (a) $\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \mathbf{x}$
   (b) $\begin{bmatrix} -3/2 & 1 \\ -1/4 & -1/2 \end{bmatrix} \mathbf{x}$
   (c) $\begin{bmatrix} -1 & -1/2 \\ 2 & -3 \end{bmatrix} \mathbf{x}$

3. Find the Laplace transform of the following functions by using a table and/or basic properties. You do not need to compute any integrals.

   (a) $f(t) = -2$
   (b) $f(t) = e^{-2t}$
   (c) $f(t) = \sin 3t$
   (d) $f(t) = te^{-3t}$
   (e) $f(t) = e^{2t} \cos 2t$

4. Sketch each of the following piecewise functions and compute their Laplace transforms.

   (a) $f(t) = \begin{cases} 0, & 0 \leq t < 4 \\ 5, & t \geq 4 \end{cases}$
   (b) $f(t) = \begin{cases} t, & 0 \leq t < 3 \\ 3, & t \geq 3 \end{cases}$

5. Engineers frequently use the *Heaviside function*, defined by

   $H(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$

   to emulate turning on a switch at a certain instance in time. Sketch the graph of the function $x(t) = e^{0.2t}$ and compute its Laplace transform, $X(s)$. On a different set of axes, sketch the graph of

   $y(t) = H(t - 3)e^{0.2t}$

   and calculate its Laplace transform, $Y(s)$. How do $X(s)$ and $Y(s)$ differ? What do you think the Laplace transform of $H(t - c)e^{0.2t}$ is, where $c$ is an arbitrary positive constant?
6. Transform the given initial value problem into an algebraic equation involving $Y(s) := \mathcal{L}(y)$, and solve for $Y(s)$.

(a) $y'' + y = \sin 4t, \quad y(0) = 0, \quad y'(0) = 1$
(b) $y'' + y' + 2y = \cos 2t + \sin 3t, \quad y(0) = -1, \quad y'(0) = 1$
(c) $y' + y = e^{-t} \sin 3t, \quad y(0) = 0$.

7. Find the inverse Laplace transform of the following functions.

(a) $Y(s) = \frac{2}{3 - 5s}$
(b) $Y(s) = \frac{1}{s^2 + 4}$
(c) $Y(s) = \frac{5s}{s^2 + 9}$
(d) $Y(s) = \frac{3}{s^2}$
(e) $Y(s) = \frac{2 - 5s}{s^2 + 9}$
(f) $Y(s) = \frac{s}{(s + 2)^2 + 4}$
(g) $Y(s) = \frac{3s + 2}{s^2 + 4s + 29}$
(h) $Y(s) = \frac{2s - 2}{(s - 4)(s + 2)}$
(i) $Y(s) = \frac{3s^2 + s + 1}{(s - 2)(s^2 + 1)}$

8. Use the Laplace transform to solve the following initial value problems.

(a) $y' - 4y = e^{-2t^2}, \quad y(0) = 1$
(b) $y'' - 9y = -2e^t, \quad y(0) = 0, \quad y'(0) = 1$.

9. Find the Laplace transform of the given functions.

(a) $3H(t - 2)$
(b) $(t - 2)H(t - 2)$
(c) $e^{2(t-1)}H(t - 1)$
(d) $H(t - \pi/4) \sin 3(t - \pi/4)$
(e) $t^2H(t - 1)$
(f) $e^{-t}H(t - 2)$

10. In this exercise, you will examine the effect of shifts in the time domain on the Laplace transform (graphically).

(a) Sketch the graph of $f(t) = \sin t$ in the time domain. Find the Laplace transform $F(s) = \mathcal{L}\{f(t)\}(s)$. Sketch the graph of $F$ in the s-domain on the interval $[0, 2]$.

(b) Sketch the graph of $g(t) = H(t - 1) \sin(t - 1)$ in the time domain. Find the Laplace transform $G(s) = \mathcal{L}\{g(t)\}(s)$. Sketch the graph of $G$ in the s-domain on the interval $[0, 2]$ on the same axes used to sketch the graph of $F$.

(c) Repeat the directions in part (b) for $g(t) = H(t - 2) \sin(t - 2)$. Explain why engineers like to say that “a shift in the time domain leads to an attenuation (scaling) in the s-domain.”