1. In the problems below, the coefficient matrix contains a parameter α. For both of these matrices, carry out the following steps.

   (i) Determine the eigenvalues in terms of α.

   (ii) Find the critical value or values of α where the qualitative nature of the phase portrait for the system changes.

   (iii) Draw a phase portrait for a value of α slight below, and for another value slightly above, each critical value.

   (iv) Draw a phase portrait when α is exactly the critical value.

   (a) $\mathbf{x}' = \begin{bmatrix} \alpha & 1 \\ -1 & \alpha \end{bmatrix} \mathbf{x}$ (b) $\mathbf{x}' = \begin{bmatrix} -1 & \alpha \\ -1 & -1 \end{bmatrix} \mathbf{x}$

2. Find the general solution for each of the given systems and draw a phase portrait. Describe the behavior of the solutions as $t \to \infty$ and as $t \to -\infty$.

   (a) $\mathbf{x}' = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \mathbf{x}$ (b) $\mathbf{x}' = \begin{bmatrix} -3/2 & 1 \\ -1/4 & -1/2 \end{bmatrix} \mathbf{x}$ (c) $\mathbf{x}' = \begin{bmatrix} -1 & -1/2 \\ 2 & -3 \end{bmatrix} \mathbf{x}$.

3. Find the Laplace transform of the following functions by using a a table and/or basic properties. You do not need to compute any intergrals.

   (a) $f(t) = -2$

   (b) $f(t) = e^{-2t}$

   (c) $f(t) = \sin 3t$

   (d) $f(t) = te^{-3t}$

   (e) $f(t) = e^{2t} \cos 2t$

4. Sketch each of the following piecewise functions and compute their Laplace transforms.

   (a) $f(t) = \begin{cases} 0, & 0 \le t < 4 \\ 5, & t \ge 4 \end{cases}$ (b) $f(t) = \begin{cases} t, & 0 \le t < 3 \\ 3, & t \ge 3 \end{cases}$

5. Engineers frequently use the *Heavyside function*, defined by

$$H(t) = \begin{cases} 0, & t < 0 \\ 1, & t \ge 0 \end{cases}$$

   to emulate turning on a switch at a certain instance in time. Sketch the graph of the function $x(t) = e^{0.2t}$ and compute its Laplace transform, $X(s)$. On a different set of axes, sketch the graph of

$$y(t) = H(t-3)e^{0.2t}$$

   and calculate its Laplace transform, $Y(s)$. How do $X(s)$ and $Y(s)$ differ? What do you think the Laplace transform of $H(t-c)e^{0.2t}$ is, where c is an arbitrary positive constant?

6. Transform the given initial value problem into an algebraic equation involving $Y(s) := \mathcal{L}(y)$, and solve for $Y(s)$.

   (a) $y'' + y = \sin 4t, \quad y(0) = 0, \quad y'(0) = 1$

   (b) $y'' + y' + 2y = \cos 2t + \sin 3t, \quad y(0) = -1, \quad y'(0) = 1$

   (c) $y' + y = e^{-t} \sin 3t, \quad y(0) = 0.$

7. Find the inverse Laplace transform of the following functions.

   (a) $Y(s) = \dfrac{2}{3-5s}$

   (b) $Y(s) = \dfrac{1}{s^2+4}$

   (c) $Y(s) = \dfrac{5s}{s^2+9}$

   (d) $Y(s) = \dfrac{3}{s^2}$

   (e) $Y(s) = \dfrac{2-5s}{s^2+9}$

   (f) $Y(s) = \dfrac{s}{(s+2)^2+4}$

   (g) $Y(s) = \dfrac{3s+2}{s^2+4s+29}$

   (h) $Y(s) = \dfrac{2s-2}{(s-4)(s+2)}$

   (i) $Y(s) = \dfrac{3s^2+s+1}{(s-2)(s^2+1)}$

8. Use the Laplace transform to solve the following initial value problems.

   (a) $y' - 4y = e^{-2t} t^2, \quad y(0) = 1$

   (b) $y'' - 9y = -2e^t, \quad y(0) = 0, \quad y'(0) = 1.$

9. Find the Laplace transform of the given functions.

   (a) $3H(t-2)$

   (b) $(t-2)H(t-2)$

   (c) $e^{2(t-1)}H(t-1)$

   (d) $H(t-\pi/4)\sin 3(t-\pi/4)$

   (e) $t^2 H(t-1)$

   (f) $e^{-t}H(t-2)$

10. In this exercise, you will examine the effect of shifts in the time domain on the Laplace transform (graphically).

    (a) Sketch the graph of $f(t) = \sin t$ in the time domain. Find the Laplace transform $F(s) = \mathcal{L}\{f(t)\}(s)$. Sketch the graph of F in the s-domain on the interval $[0, 2]$.

    (b) Sketch the graph of $g(t) = H(t-1)\sin(t-1)$ in the time domain. Find the Laplace transform $G(s) = \mathcal{L}\{g(t)\}(s)$. Sketch the graph of G in the s-domain on the interval $[0, 2]$ on the same axes used to sketch the graph of F.

    (c) Repeat the directions in part (b) for $g(t) = H(t-2)\sin(t-2)$. Explain why engineers like to say that "a shift in the time domain leads to an attenuation (scaling) in the s-domain."