MthSc 208: Differential Equations (Summer I, 2013)

In-class Worksheet 4b: Systems of differential equations (real eigenvalues)

NAME:

Consider the system of differential equations: $\begin{cases} x_1' = -2x_1 + x_2 - 12, & x_1(0) = 5 \\ x_2' = 4x_1 + x_2 - 18, & x_2(0) = 20 \end{cases}$

1. Write this in matrix form, $\mathbf{x}' = \mathbf{A}\mathbf{x} + \mathbf{b}$, and find the steady-state solution, $\mathbf{x}_{ss}$.

2. Make a change of variables to transform the system into a homogeneous system.

3. Solve the homogeneous system. (Recall from Worksheet 6 that the eigenvalues of $\mathbf{A}$ are $\lambda_1 = -3$, $\lambda_2 = 2$, with eigenvectors $\mathbf{v}_1 = (-1, 1)$ and $\mathbf{v}_2 = (1, 4)$.)

4. Solve the solution to the original system of differential equations, by writing $\mathbf{x} = \mathbf{x}_h + \mathbf{x}_{ss}$.

5. Sketch the phases portrait of the homogeneous system, and the inhomogeneous system. Also sketch the particular solutions satisfying the given initial conditions.