MthSc 208: Differential Equations (Summer I, 2013)

In-class Worksheet 4d: Systems of differential equations (repeated eigenvalues)

NAME:

Consider the system of differential equations: $\begin{cases} x_1' = -x_1 - x_2 \\ x_2' = x_1 - 3x_2 \end{cases}$

1. Write this in matrix form, $\mathbf{x}' = \mathbf{A}\mathbf{x} + \mathbf{b}$.

2. Knowing that $\mathbf{A}$ has a repeated eigenvalue, $\lambda_{1,2} = -2$, and one eigenvector, $\mathbf{v}_1 = (1, 1)$, write down a solution $\mathbf{x}_1(t)$ to $\mathbf{x}' = \mathbf{A}\mathbf{x}$.

3. To find a second solution, assume that $\mathbf{x}_2(t) = te^{\lambda t}\mathbf{v} + e^{\lambda t}\mathbf{w}$. Plug this back into $\mathbf{x}' = \mathbf{A}\mathbf{x}$ and equate coefficients (of $te^{-\lambda t}$ and $e^{\lambda t}$) to get a system of two equations, involving $\mathbf{v}$, $\mathbf{w}$, and $\mathbf{A}$.

4. Solve for $\mathbf{v}$ by inspection. Plug this back into the second equation and solve for $\mathbf{w}$ (it will involve a constant, C).

5. Using what you got for $\mathbf{v}(t)$ and $\mathbf{w}(t)$, write down a solution $\mathbf{x}_2(t)$ that is not a scalar multiple of $\mathbf{x}_1$. (Pick the simplest value of C that works.)

6. Write down the general solution, $\mathbf{x}(t)$.

7. As $t \to \infty$, which of the three terms of $\mathbf{x}(t)$ "goes to zero slower"? Use this intuition to sketch the phase portrait.