1. Prove that both of the following series conditionally converge:

   (a) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{\ln n}{n}$ (b) $\displaystyle\sum_{n=1}^{\infty}\frac{\sin n}{n}$.

2. Suppose $\displaystyle\lim_{n\to\infty} na_n = A \neq 0$. Prove that $\displaystyle\sum_{n=1}^{\infty} a_n$ diverges.

3. Determine the convergence/divergence of the following series:

   (a) $1+\frac{1}{2}-\frac{1}{3}+\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\frac{1}{7}+\frac{1}{8}-\frac{1}{9}+\cdots$.

   (b) $1-\frac{1}{2}-\frac{1}{3}+\frac{1}{4}+\frac{1}{5}-\frac{1}{6}-\frac{1}{7}+\frac{1}{8}+\frac{1}{9}+\cdots$.

4. If $\displaystyle\sum_{n=1}^{\infty} a_n$ converges conditionally, prove there exist rearrangements a_n' and $\widetilde{a}_n$ such that

$$\sum_{n=1}^{\infty} a_n' = \infty \qquad \text{and} \qquad \sum_{n=1}^{\infty} \widetilde{a}_n = -\infty\,.$$

5. Prove the following:

   (a) $\displaystyle\sum_{n=1}^{\infty}|a_n|^2<\infty$ and $\displaystyle\sum_{n=1}^{\infty}|b_n|^2<\infty \implies \sum_{n=1}^{\infty}a_nb_n$ converges absolutely.

   (b) $\displaystyle\sum_{n=1}^{\infty}|a_n|<\infty$ and $\displaystyle\sum_{n=1}^{\infty}b_n<\infty \implies \sum_{n=1}^{\infty}a_nb_n$ converges absolutely.

6. Prove that $\ell^1 \subsetneq \ell^2 \subsetneq \ell^\infty$.

7. Prove that the following are normed linear spaces:

   (a) $(\ell^p, ||\cdot||_p)$ for $p=1,2,\infty$.

   (b) $(\mathcal{B}[0,1], ||\cdot||_\infty)$, $(\mathcal{R}[0,1], ||\cdot||_\infty)$, $(\mathcal{C}[0,1], ||\cdot||_\infty)$.

8. Find the pointwise limit of each of the following sequences of functions:

   (a) $f_n(x) = \dfrac{\sin nx}{1+nx}$ on $[0,\infty)$ (b) $f_n(x) = nxe^{-nx^2}$ on $\mathbb{R}$.

9. Let $f_n\colon \mathbb{N}\to\mathbb{R}$ defined by $f_n(m)=\frac{n}{m+n}$. Show that $\displaystyle\lim_{m\to\infty}\lim_{n\to\infty} f_n(m) \neq \lim_{n\to\infty}\lim_{m\to\infty} f_n(m)$.