

Read the following, which can all be found either in the textbook or on the course website.

- Chapters 1 & 2 of *Visual Group Theory* (VGT).
- VGT Exercises 1.1–1.4, 1.8–1.12, 2.13–2.17.
- The article *Group Think* by Steven Strogatz, which appeared in the NY Times in 2010.
- Francis Su’s essay on good mathematical writing.


Write up solutions to the following exercises.

1. Answer the following questions after reading Francis Su’s essay on good mathematical writing.
 - (a) What is a good rule of thumb for what you should assume of your audience as you write your homework sets?
 - (b) Is chalkboard writing formal or informal writing?
 - (c) Why is the proof by contradiction on page 3 not really a proof by contradiction?
 - (d) Name three things a lazy writer would do that a good writer would not.
 - (e) What’s the difference in meaning between these three phrases?

“Let $A = 12.$ ”

“So $A = 12.$ ”

“ $A = 12.$ ”

2. Given a regular n -gon, let r be a rotation of it by $2\pi/n$ radians. This time, assume that we are not allowed to flip over the n -gon. These n actions form a group denoted $C_n = \langle r \rangle = \{e, r, r^2, \dots, r^{n-1}\}$.
 - (a) Draw a Cayley diagram for C_n for $n = 4$, $n = 5$, and $n = 6$.
 - (b) For $n = 4, 5, 6$, find all minimal generating sets of C_n .
 - (c) Make a conjecture of what integers k does $C_n = \langle r^k \rangle$ for a general fixed integer n .
3. As we saw in lecture, the six symmetries of an equilateral triangle  form a group denoted $D_3 = \{e, r, r^2, f, rf, r^2f\}$, where r is a 120° clockwise rotation and f is a flip about a vertical axis (which fixes the top corner). Since r and f suffice to generate all six of these symmetries, we write $D_3 = \langle r, f \rangle$.
 - (a) Let g be the reflection of the triangle that fixes the lower-left corner. Which of the six actions in D_3 is g equal to? Which action is fg ?
 - (b) Write all 6 actions of D_3 using only f and g . Draw a Cayley diagram using f and g as generators.
 - (c) To generate D_3 , we need at least 2 actions. It is not difficult to show that if we have 3 generators, then one of them is unnecessary. Find all *minimal* generating sets of $D_3 = \{e, r, r^2, f, rf, r^2f\}$; note that all of them should have exactly two actions. Do not use g in this list.

4. The eight symmetries of a square $\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$ form a group denoted D_4 . Let r be a 90° clockwise rotation and f a horizontal flip (that is, about a vertical axis). It is not difficult to show that $D_4 = \langle r, f \rangle$.
- (a) Write all 8 actions of D_4 using r and f and draw a Cayley diagram using these two actions as generators.
 - (b) Let g be the reflection of the square that fixes the lower-left and upper-right corner. Which of the six actions in D_3 is g equal to? Which action is fg ?
 - (c) Draw a Cayley diagram of D_4 using f and g as generators.
 - (d) Find all *minimal* generating sets of D_4 . [*Hint*: There are 12.]
5. Pick any integer and consider this set of actions: adding any integer to the one you choose. This is an infinite set of actions; we might name them like “add 1” and “add -4210 ,” etc. This is a group. Find all minimal generating sets. Sketch a Cayley graph for this group using one of these minimal generating sets.