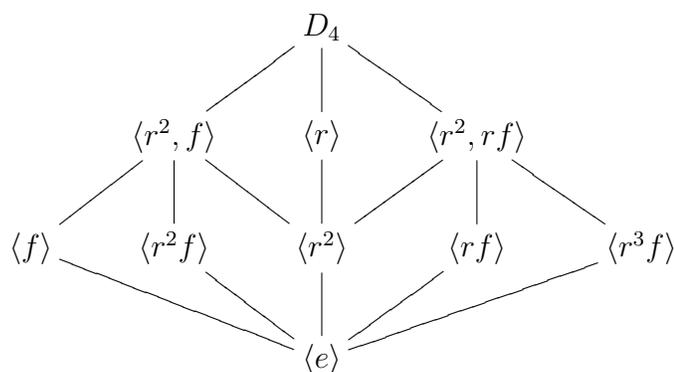


Read the following, which can all be found either in the textbook or on the course website.

- Chapters 6 of *Visual Group Theory* (VGT).
- VGT Exercises 6.6–6.9, 6.12, 6.17–6.20, 6.28–6.30.

Write up solutions to the following exercises.

1. Consider the subgroups  $H = \langle (1\ 2\ 3) \rangle$  and  $K = \langle (1\ 2)(3\ 4) \rangle$  of the alternating group  $G = A_4$ . Carry out the following steps for both of these subgroups.
  - (a) Write  $G$  as a disjoint union of the subgroup's left cosets.
  - (b) Write  $G$  as a disjoint union of the subgroup's right cosets.
  - (c) Determine whether the subgroup is normal in  $G$ .
2. The subgroup lattice of  $D_4$  is shown below:



For each of the 10 subgroups of  $D_4$ , determine whether it is normal. Justify each yes or no answer.

3. Consider a chain of subgroups  $K \leq H \leq G$ .
  - (a) Prove or disprove: If  $K \triangleleft H \triangleleft G$ , then  $K \triangleleft G$ .
  - (b) Prove or disprove: If  $K \triangleleft G$ , then  $K \triangleleft H$ .

4. The *center* of a group  $G$  is the set

$$\begin{aligned} Z(G) &= \{z \in G \mid gz = zg, \forall g \in G\} \\ &= \{z \in G \mid gzg^{-1} = z, \forall g \in G\}. \end{aligned}$$

Prove that  $Z(G)$  is a subgroup of  $G$ , and that it is normal in  $G$ .

5. Let  $H$  be a subgroup of  $G$ . Given two fixed elements  $a, b \in G$ , define the sets

$$aHbH = \{ah_1bh_2 \mid h_1, h_2 \in H\} \quad \text{and} \quad abH = \{abh \mid h \in H\}.$$

Prove that if  $H \triangleleft G$ , then  $aHbH = abH$ .

6. Let  $H$  be a subgroup of an abelian group  $G$ . Prove that  $H$  and  $G/H$  are both abelian.