

## Math 2080: Differential Equations

### Worksheet 1.1: What is a differential equation?

NAME:

1. Consider an investment that grows at a constant interest rate of 5%, compounded continuously. Suppose that initially, the investment is worth \$1000. If  $P(t)$  is the value of the investment, write out an *initial value problem* (IVP) – a differential equation with an initial condition – that  $P(t)$  satisfies and sketch the solutions.
  
  
  
  
  
  
  
  
  
  
2. Consider an investment that grows at a constant interest rate of 5%, compounded continuously. Suppose that initially, the investment is growing at a *rate* of \$100/year. If  $P(t)$  is the value of the investment, write out an IVP that  $P(t)$  satisfies and sketch the solutions.
  
  
  
  
  
  
  
  
  
  
3. The mass  $m(t)$  of a radioactive substance decays at a rate proportional to the amount remaining. Suppose there are 100 grams initially. Write out an IVP that  $m(t)$  satisfies and sketch the solutions.

4. The temperature  $T(t)$  of a cup of coffee cools at a rate proportional to the difference in its temperature with the ambient room temperature (say,  $70^\circ$ ). Suppose that the coffee is initially  $190^\circ$ .

(a) Write out an initial value problem that  $T(t)$  satisfies and sketch the solutions.

(b) Let  $y(t) = T(t) - 70$ , which represents the number of degrees that the coffee is above or below the room temperature. Substituting  $T = y + 70$  back into your IVP from the previous part to get a much simpler IVP in terms of the function  $y(t)$  and sketch the solutions.