

## Math 2080: Differential Equations

### Worksheet 3.4: Simple harmonic motion

**NAME:**

When a 3 kg mass is hung from a spring, the spring is displaced by 1 m. Now, suppose that the mass is hanging from equilibrium and then given a sharp jolt downwards so its initial velocity is 1 m/sec. We will set up and solve an initial value problem that models this.

- (a) Sketch this mass-spring system, before and after the mass is placed on the spring. (Let  $x = 0$  be the height of the spring without the mass).
- (b) At equilibrium, the spring force  $kx_0$  equals the gravitational force,  $mg$ , in magnitude. Use this to solve for the spring constant  $k$ .
- (c) Newton's 2<sup>nd</sup> law tells us that  $F = mx''$ , which is equal to the sum of the forces (gravitational and spring). Write down a second-order differential equation that models this. Include both initial conditions,  $x(0)$  and  $x'(0)$ .

(d) Find the general solution to this ODE. Is there a steady-state solution? If so, describe it.

(e) Solve the initial value problem.