Math 2080: Differential Equations Worksheet 3.5: Damped harmonic motion

NAME:

1. For the following exercises, rewrite the given function in the form

$$y = A\cos(\omega t - \phi) = A\cos\left(\omega\left(t - \frac{\phi}{\omega}\right)\right),$$

and then plot the graph of this function.

(a)
$$y = 3\cos 2t + 4\sin 2t$$

(b)
$$y = 3\cos 2t - 4\sin 2t$$

(c)
$$y = -3\cos 2t - 4\sin 2t$$

2. Consider the undamped oscillator

$$mx'' + kx = 0,$$
 $x(0) = x_0,$ $x'(0) = v_0.$

Write the general solution of this initial value problem in the form $x(t) = a\cos\omega t + b\sin\omega t$ (i.e., determine a, b, and ω .), and then write it in the form $x(t) = A\cos(\omega t - \phi)$ (i.e., determine A).

3. The function $x(t) = \cos 6t - \cos 7t$ has mean frequency $\bar{\omega} = 13/2$ and half difference $\delta = 1/2$. Thus,

$$\cos 6t - \cos 7t = \cos \left(\frac{13}{2} - \frac{1}{2}\right)t - \cos \left(\frac{13}{2} + \frac{1}{2}\right)t = 2\sin \frac{1}{2}t \sin \frac{13}{2}t.$$

Use a computer or calculator to plot both $f(t) = \cos 6t - \cos 7t$ and the "envelope" $g(t) = 2\sin \frac{1}{2}t$ on the same set of axes.

4. Let $\omega_0 = 11$. Use a computer to plot the graph of the function

$$x(t) = \frac{\cos \omega t - \cos \omega_0 t}{\omega_0^2 - \omega^2}$$

for $\omega=9,\,10,\,10.5,\,10.9,$ and 10.99 on the time interval [0,24]. Explain how these solutions approach the resonance solution as $\omega\to\omega_0$.