Math 2080: Differential Equations
Worksheet 3.6: Variation of parameters

NAME:

Consider the inhomogeneous differential equation $y'' + 9y = \sec 3t$.

(a) Solve the related homogeneous equation, $y_h'' + 9y_h = 0$, by inspection.

(b) If $y_h(t) = C_1 y_1(t) + C_2 y_2(t)$ is your answer to the previous part, then assume that there is a particular solution to the original ODE of the form $y_p = v_1 y_1 + v_2 y_2$. Compute y_p'.

(c) The y_p' from the previous part should have four terms; one each with v_1, v_2, v_1', and v_2'. Eliminate the two terms involving v_1' and v_2' by setting their sum equal to zero. This leaves just two terms in y_p'. Now compute y_p''.

(d) Plug $y_p = v_1 y_1 + v_2 y_2$ and y_p'' from Part (c) back into the original ODE. Things should cancel, leaving you with an equation involving v_1' and v_2' (no v_1 or v_2).

(e) Write a system of two first order ODEs involving v_1' and v_2'. One equation comes from Part (c) and the other from Part (d). Write this in matrix form as well.

(f) Solve for v_1' and v_2' by multiplying your two equations by suitible functions and and or subtracting them to eliminate the other.

(g) Integrate to find v_1 and v_2 (feel free to use a computer). Write the general solution to the original ODE.