Math 2080: Differential Equations
Worksheet 3.9: The method of Frobenius

NAME:

In this problem, you will solve $3xy'' + y' + y = 0$ using the method of Frobenius.

(a) Write this equation as $y'' + P(x)y' + Q(x)y = 0$. Determine whether $x_0 = 0$ is an ordinary, regular singular, or irregular singular point.

(b) Assume there is a solution of the form $y(x) = \sum_{n=0}^{\infty} a_n x^{n+r}$. Plug this back into the ODE.

(c) Factor out x^r, and then shift indices so you can combine everything into a single sum. Don't forget to "pick off" the $n = -1$ term to get the indicial equation.

(d) Solve the indicial equation for r, and determine the recursion formula for the a_i's.

(e) For each value of r, write out a generalized power series and the recursion formula for the coefficients.

(f) What is the radius of convergence of this solution? (Hint: First compute the radius of convergence of $xP(x)$ and $x^2Q(x)$.)