

## Math 2080: Differential Equations

### Worksheet 4.3: Mixing with two tanks

**NAME:**

Tank  $A$  contains 10 gallons of a solution in which 5 oz of salt are dissolved. Tank  $B$  contains 20 gallons in which 6 oz of salt are dissolved. Salt water with a concentration of 2 oz/gal flows into each tank at a rate of 4 gal/min. The fully mixed solution drains from Tank  $A$  at a rate of 3 gal/min and from Tank  $B$  at a rate of 5 gal/min. Solution flows from Tank  $A$  to Tank  $B$  at a rate of 1 gal/min. Let  $\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$ , where  $x_1(t)$  (respectively,  $x_2(t)$ ) is the amount of salt in Tank  $A$  (resp., Tank  $B$ ) after time  $t$ .

- (a) Write down a system of ODEs (including the initial condition  $\mathbf{x}(0)$ ) that models this situation, and write it in matrix form:  $\mathbf{x}' = \mathbf{A}\mathbf{x} + \mathbf{b}$ ,  $\mathbf{x}(0) = \mathbf{c}$ .

- (b) What is the steady-state solution,  $\mathbf{x}_{ss}$ ?

(c) If  $\mathbf{x}_{ss} = \begin{bmatrix} a \\ b \end{bmatrix}$ , then change variables by setting  $y_1 = x_1 - a$  and  $y_2 = x_2 - b$ . Plug  $y_1$  and  $y_2$  back into the system to get a related system in  $\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ . Don't forget the initial condition,  $\mathbf{y}(0)$ .