Math 2080: Differential Equations
Worksheet 4.5: Phase portraits with real eigenvalues

NAME:

Suppose the eigenvalues and eigenvectors of a 2×2 matrix $\boldsymbol{A}$ are given. Write the general solution to the system $\boldsymbol{x}' = \boldsymbol{A}\boldsymbol{x}$. Then, sketch the phase portrait (the graph x_2 vs. x_1). Make sure that your sketch is accurate enough that it is clear which way the solution curves "bend", if applicable. Also, you should clearly distinguish between e.g., a line of slope 2 and a line of slope $1/2$.

1. $\lambda_1 = -2$, $\lambda_2 = 2$, $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$.

2. $\lambda_1 = -2$, $\lambda_2 = -3$, $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$.

3. $\lambda_1 = -2$, $\lambda_2 = -30$, $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$.

4. $\lambda_1 = 2$, $\lambda_2 = 3$, $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$.

5. $\lambda_1 = 0.2$, $\lambda_2 = 3$, $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$.

6. $\lambda_1 = 0$, $\lambda_2 = 3$, $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$.